4. It is desired to estimate the value of a DC level A in WGN or

$$x[n] = A + w[n] \qquad n = 0, 1, \ldots, N - 1$$

where $w[n]$ is zero mean and uncorrelated, and each sample has variance $\sigma^2 = 1$. Consider the two estimators

$$\hat{A} = \frac{1}{N} \sum_{n=0}^{N-1} x[n]$$

$$\check{A} = \frac{1}{N+2} \left(2x[0] + \sum_{n=1}^{N-2} x[n] + 2x[N-1] \right).$$

Which one is better? Does it depend on the value of A?

5. For the same data set as in Problem 1.4 the following estimator is proposed:

$$\hat{A} = \begin{cases} x[0] & \frac{A^2}{\sigma^2} = A^2 > 1000 \\ \frac{1}{N} \sum_{n=0}^{N-1} x[n] & \frac{A^2}{\sigma^2} = A^2 \leq 1000. \end{cases}$$

The rationale for this estimator is that for a high enough signal-to-noise ratio (SNR) or A^2/σ^2, we do not need to reduce the effect of noise by averaging and hence can avoid the added computation. Comment on this approach.