

Hw 1 Solution

4) As shown in text $E(\hat{A}) = A$. Also,

$$E(\check{A}) = \frac{1}{N+2} (2A + (N-2)A + 2A) = A$$

Also, we know that $\text{var}(\hat{A}) = \sigma^2/N = 1/N$ and

$$\begin{aligned}\text{var}(\check{A}) &= \frac{1}{(N+2)^2} \left[4\sigma^2 + \sum_{n=1}^{N-2} \sigma^2 + 4\sigma^2 \right] \\ &= \frac{N+6}{(N+2)^2} \sigma^2 = \frac{N+6}{(N+2)^2}\end{aligned}$$

$$\begin{aligned}\text{var}(\check{A}) - \text{var}(\hat{A}) &= \frac{N+6}{(N+2)^2} - \frac{1}{N} \\ &= \frac{N(N+6) - (N+2)^2}{N(N+2)^2} \\ &= \frac{2N-4}{N(N+2)^2} > 0 \text{ for } N > 2\end{aligned}$$

Hence, both estimators yield the correct value on the average but \hat{A} has less variance. Conclusion is the same for any value of A .

5) \hat{A} is not an estimator since to implement it requires knowledge of A (to determine the SNR).