9) Using (3.32)

$$I(A) = \left(\frac{\partial \underline{\mu}(A)}{\partial A}\right)^T \underline{C}^{-1} \frac{\partial \underline{\mu}(A)}{\partial A}$$

$$\underline{\mu}(A) = [A \; A]^T \Rightarrow \frac{\partial \underline{\mu}(A)}{\partial A} = \underline{1}$$

$$\operatorname{var}(\hat{A}) \geq \frac{1}{\underline{1}^T \underline{C}^{-1} \underline{1}}$$ (or use approach of Prob 3.5)

$$\underline{C}^{-1} = \frac{\frac{1}{\sigma^2}\begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix}}{1-\rho^2}$$

$$\Rightarrow \operatorname{var}(\hat{A}) \geq \frac{\sigma^2(1-\rho^2)}{2-2\rho} = \frac{\sigma^2}{2}(1+\rho)$$

If $\rho = 0$, $\text{var}(\hat{A}) \geq \sigma^2/2$, as expected. If $\rho \to 1$, $\text{var}(\hat{A}) \geq \sigma^2$. This is the same bound as for one sample and occurs because as $\rho \to 1$, $w[0]$ and $w[1]$ will be equal. Hence, we have only one independent data sample. If $\rho \to -1$, $\text{var}(\hat{A}) \geq 0$ and in fact, in this case $w[0] = -w[1]$. Thus,

$$\hat{A} = \frac{1}{2}(x[0] + x[1])$$

$$= \frac{1}{2}(A + w[0] + A - w[0]) = A$$

for any realization of the noise samples. Additivity property of Fisher information only holds for independent samples. In this example we could have

$$i(A) \leq I(A) < \infty \, i(A)$$

where $i(A) = 1/\sigma^2$.