**Hw 5 Solution**

3) a) $$p(\underline{x}; \mu) = \prod_{n=0}^{N-1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x[n]-\mu)^2}$$

$$= \frac{1}{(2\pi)^{N/2}} e^{-\frac{1}{2}\sum_{n=0}^{N-1}(x[n]-\mu)^2}$$

To maximize $p$, we minimize $\sum_n (x[n]-\mu)^2$. Since it is a quadratic in $\mu$, differentiation produces a global minimum.

$$\Rightarrow \quad \sum_n (x[n]-\mu) = 0 \quad \Rightarrow \quad \hat{\mu} = \bar{x}$$

(This is just a DC level, $\mu$, in WGN).

b) $$p(\underline{x}; \lambda) = \begin{cases} \lambda^N e^{-\lambda \sum_n x[n]} & \text{all } x[n] > 0 \\ 0 & \text{otherwise} \end{cases}$$

Assuming all $x[n] > 0$ we have

$$p = \lambda^N e^{-\lambda N \bar{x}}$$

$$\frac{dp}{d\lambda} = N\lambda^{N-1} e^{-\lambda N \bar{x}} + \lambda^N(-N\bar{x}) e^{-\lambda N \bar{x}} = 0$$

$$\Rightarrow \quad \hat{\lambda} = 1/\bar{x}$$

To verify that $\hat{\lambda}$ yields the global maximum we can consider $\ln p$, which is a monotonic function of $p$.

$$\ln p = N \ln \lambda - \lambda N \bar{x}$$

$$\frac{d \ln p}{d\lambda} = N/\lambda - N\bar{x}$$

$$\frac{d^2 \ln p}{d\lambda^2} = -N/\lambda^2 < 0 \quad \text{for all } \lambda$$

$\Rightarrow$ $\ln p$ is concave function

$\Rightarrow$ $\hat{\lambda}$ is global maximum solution

This result is reasonable since the mean of $x$ is $1/\lambda$.