

UNIVERSITY OF ARKANSAS AT LITTLE  
ROCK

Department of Systems Engineering

SYEN 3314 Probability and Random Signals - Summer 2009

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**Final Exam** - Monday, June 29, 2009

- This is a closed book exam.
- Calculators are not allowed.
- There are 10 problems on the exam plus one extra credit (or bonus) problem.
- The problems are not in order of difficulty. We recommend that you read through all the problems, then do the problems in whatever order suits you best.
- A correct answer does not guarantee full credit, and a wrong answer does not guarantee loss of credit. You should clearly but concisely indicate your reasoning and show all relevant work. Your grade on each problem will be based on our assessment of your level of understanding as reflected by what you have written in the space provided.
- Please be neat and box your final answer, we cannot grade what we cannot decipher.

**Name**

## Problem 1

Monitor three consecutive phone calls going through a telephone switching office. Classify each one as a voice call ( $v$ ) if someone is speaking, or a data call ( $d$ ) if the call is carrying a modem or fax signal. Your observation is a sequence of three letters (each one of them is either  $v$  or  $d$ ). For example, three voice calls correspond to  $vvv$ . The outcomes  $vvv$  and  $ddd$  have **probability 0.2** whereas each of the other outcomes  $vvd, vdv, vdd, dvv, dvd$  and  $ddv$  has **probability 0.1**. Count the number of voice calls  $N_v$  in the three calls you have observed. Consider the four events  $N_v = 0, N_v = 1, N_v = 2, N_v = 3$ . Describe in words and also calculate the following probabilities:

1.  $P[N_v = 2]$
2.  $P[N_v \geq 1]$
3.  $P[\{vvd\} | N_v = 2]$
4.  $P[\{ddv\} | N_v = 2]$
5.  $P[N_v = 2 | N_v \geq 1]$
6.  $P[N_v \geq 1 | N_v = 2]$

## Problem 2

The number of database queries processed by a computer in any 10-second interval is a Poisson random variable,  $K$ , with  $\alpha = 5$  queries. The Poisson distribution is given by

$$P_K(k) = \begin{cases} \alpha^k e^{-\alpha} / k!, & k = 0, 1, 2, \dots; \\ 0, & \text{otherwise.} \end{cases}$$

1. What is the probability that there will be no queries processed in a 10-second interval?
2. What is the probability that **at least** two queries will be processed in a 2-second interval?

### Problem 3

The random variable  $X$  has PMF

$$P_X(x) = \begin{cases} c/x, & x = 2, 4, 8; \\ 0, & \text{otherwise.} \end{cases}$$

1. What is the value of the constant  $c$ ?
2. What is  $P[X = 4]$ ?
3. What is  $P[X < 4]$ ?
4. What is  $P[3 \leq X \leq 9]$ ?

## Problem 4

The random variable  $X$  has CDF

$$F_X(x) = \begin{cases} 0, & x < -3; \\ 0.4, & -3 \leq x < 5; \\ 0.8, & 5 \leq x < 7; \\ 1, & x \geq 7. \end{cases}$$

1. Draw a graph of the CDF
2. Find  $P_X(x)$ , the PMF of  $X$ .

## Problem 5

The probability density function of the random variable  $Y$  is

$$f_Y(y) = \begin{cases} 3y^2/2, & -1 \leq y \leq 1; \\ 0, & \text{otherwise.} \end{cases}$$

Sketch the PDF and find the following:

1. The expected value  $E[Y]$
2. The second moment  $E[Y^2]$
3. The variance  $Var[Y]$
4. The standard deviation  $\sigma_Y$

## Problem 6

The random variable  $W$  has CDF

$$F_W(w) = \begin{cases} 0, & x < -5; \\ (w + 5)/8, & -5 \leq x < -3; \\ 1/4, & -3 \leq x < 3; \\ \frac{1}{4} + 3\frac{(w-3)}{8}, & 3 \leq x < 5. \\ 1, & w \geq 5. \end{cases}$$

1. What is  $P[W \leq 4]$
2. What is  $P[-2 < W \leq 2]$
3. What is  $P[W > 0]$
4. What is the value of  $a$  such that  $P[W \leq a] = 1/2$ ?

## Problem 7

$X$  is the Gaussian  $(0, 1)$  random variable and  $Y$  is the Gaussian  $(0, 2)$  random variable.

1. What is  $P[-1 < X \leq 1]$
2. What is  $P[-1 < Y \leq 1]$
3. What is  $P[X > 3.5]$
4. What is  $P[Y > 3.5]$

The table of the standard normal CDF is given at the beginning of the exam sheet.



## Problem 8

The joint PMF  $P_{Q,G}(q, g)$  for random variables  $Q$  and  $G$  is given in the following table

$P_{Q,G}(q, g)$	$g = 0$	$g = 1$	$g = 2$	$g = 3$
$q = 0$	0.06	0.18	0.24	0.12
$q = 1$	0.04	0.12	0.16	0.08

Find

1.  $P[Q = 0]$
2.  $P[Q = G]$
3.  $P[G > 1]$
4.  $P[G > Q]$
5. The marginal PMF of  $G$ ,  $P_G(g)$
6. The marginal PMF of  $Q$ ,  $P_Q(q)$ .

## Problem 9

Random variables  $X$  and  $Y$  have the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 1/2, & -1 \leq x \leq y \leq 1; \\ 0, & \text{otherwise.} \end{cases}$$

1. Sketch the region of non-zero probability
2. What is  $P[X > 0]$
3. What is the marginal PDF  $f_X(x)$
4. What is  $E[X]$

## Problem 10

Random variables  $X$  and  $Y$  have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} c, & 0 \leq x \leq 5, 0 \leq y \leq 3; \\ 0, & \text{otherwise.} \end{cases}$$

Find

1. the constant  $c$
2.  $P[2 \leq X < 3, 1 \leq Y < 3]$

## Extra credit problem worth 10 points

A professor pays 25 cents for each blackboard error made in lecture to the student who points out the error. In a career of  $n$  years filled with blackboard errors, the total amount in dollars paid can be approximated by a Gaussian random variable  $Y_n$  with expected value  $40n$  and variance  $100n$ .

1. What is the probability that  $Y_{20}$  exceeds 1000?
2. How many years  $n$  must the professor teach in order that  $P[Y_n > 1000] > 0.99$ ?

$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.97725	2.50	0.99379
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.01	0.97778	2.51	0.99396
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.02	0.97831	2.52	0.99413
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.03	0.97882	2.53	0.99430
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.04	0.97932	2.54	0.99446
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.05	0.97982	2.55	0.99461
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.06	0.98030	2.56	0.99477
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.07	0.98077	2.57	0.99492
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.08	0.98124	2.58	0.99506
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.09	0.98169	2.59	0.99520
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.10	0.98214	2.60	0.99534
0.11	0.5438	0.61	0.7291	1.11	0.8665	1.61	0.9463	2.11	0.98257	2.61	0.99547
0.12	0.5478	0.62	0.7324	1.12	0.8686	1.62	0.9474	2.12	0.98300	2.62	0.99560
0.13	0.5517	0.63	0.7357	1.13	0.8708	1.63	0.9484	2.13	0.98341	2.63	0.99573
0.14	0.5557	0.64	0.7389	1.14	0.8729	1.64	0.9495	2.14	0.98382	2.64	0.99585
0.15	0.5596	0.65	0.7422	1.15	0.8749	1.65	0.9505	2.15	0.98422	2.65	0.99598
0.16	0.5636	0.66	0.7454	1.16	0.8770	1.66	0.9515	2.16	0.98461	2.66	0.99609
0.17	0.5675	0.67	0.7486	1.17	0.8790	1.67	0.9525	2.17	0.98500	2.67	0.99621
0.18	0.5714	0.68	0.7517	1.18	0.8810	1.68	0.9535	2.18	0.98537	2.68	0.99632
0.19	0.5753	0.69	0.7549	1.19	0.8830	1.69	0.9545	2.19	0.98574	2.69	0.99643
0.20	0.5793	0.70	0.7580	1.20	0.8849	1.70	0.9554	2.20	0.98610	2.70	0.99653
0.21	0.5832	0.71	0.7611	1.21	0.8869	1.71	0.9564	2.21	0.98645	2.71	0.99664
0.22	0.5871	0.72	0.7642	1.22	0.8888	1.72	0.9573	2.22	0.98679	2.72	0.99674
0.23	0.5910	0.73	0.7673	1.23	0.8907	1.73	0.9582	2.23	0.98713	2.73	0.99683
0.24	0.5948	0.74	0.7704	1.24	0.8925	1.74	0.9591	2.24	0.98745	2.74	0.99693
0.25	0.5987	0.75	0.7734	1.25	0.8944	1.75	0.9599	2.25	0.98778	2.75	0.99702
0.26	0.6026	0.76	0.7764	1.26	0.8962	1.76	0.9608	2.26	0.98809	2.76	0.99711
0.27	0.6064	0.77	0.7794	1.27	0.8980	1.77	0.9616	2.27	0.98840	2.77	0.99720
0.28	0.6103	0.78	0.7823	1.28	0.8997	1.78	0.9625	2.28	0.98870	2.78	0.99728
0.29	0.6141	0.79	0.7852	1.29	0.9015	1.79	0.9633	2.29	0.98899	2.79	0.99736
0.30	0.6179	0.80	0.7881	1.30	0.9032	1.80	0.9641	2.30	0.98928	2.80	0.99744
0.31	0.6217	0.81	0.7910	1.31	0.9049	1.81	0.9649	2.31	0.98956	2.81	0.99752
0.32	0.6255	0.82	0.7939	1.32	0.9066	1.82	0.9656	2.32	0.98983	2.82	0.99760
0.33	0.6293	0.83	0.7967	1.33	0.9082	1.83	0.9664	2.33	0.99010	2.83	0.99767
0.34	0.6331	0.84	0.7995	1.34	0.9099	1.84	0.9671	2.34	0.99036	2.84	0.99774
0.35	0.6368	0.85	0.8023	1.35	0.9115	1.85	0.9678	2.35	0.99061	2.85	0.99781
0.36	0.6406	0.86	0.8051	1.36	0.9131	1.86	0.9686	2.36	0.99086	2.86	0.99788
0.37	0.6443	0.87	0.8078	1.37	0.9147	1.87	0.9693	2.37	0.99111	2.87	0.99795
0.38	0.6480	0.88	0.8106	1.38	0.9162	1.88	0.9699	2.38	0.99134	2.88	0.99801
0.39	0.6517	0.89	0.8133	1.39	0.9177	1.89	0.9706	2.39	0.99158	2.89	0.99807
0.40	0.6554	0.90	0.8159	1.40	0.9192	1.90	0.9713	2.40	0.99180	2.90	0.99813
0.41	0.6591	0.91	0.8186	1.41	0.9207	1.91	0.9719	2.41	0.99202	2.91	0.99819
0.42	0.6628	0.92	0.8212	1.42	0.9222	1.92	0.9726	2.42	0.99224	2.92	0.99825
0.43	0.6664	0.93	0.8238	1.43	0.9236	1.93	0.9732	2.43	0.99245	2.93	0.99831
0.44	0.6700	0.94	0.8264	1.44	0.9251	1.94	0.9738	2.44	0.99266	2.94	0.99836
0.45	0.6736	0.95	0.8289	1.45	0.9265	1.95	0.9744	2.45	0.99286	2.95	0.99841
0.46	0.6772	0.96	0.8315	1.46	0.9279	1.96	0.9750	2.46	0.99305	2.96	0.99846
0.47	0.6808	0.97	0.8340	1.47	0.9292	1.97	0.9756	2.47	0.99324	2.97	0.99851
0.48	0.6844	0.98	0.8365	1.48	0.9306	1.98	0.9761	2.48	0.99343	2.98	0.99856
0.49	0.6879	0.99	0.8389	1.49	0.9319	1.99	0.9767	2.49	0.99361	2.99	0.99861

Figure 1: