Bayesian approach for reconstruction of moving brain dipoles

Petia Georgieva[1], Nidhal Bouaynaya[2], Lyudmila Mihaylova[3], and Filipe Silva[1]

[1] University of Aveiro, Aveiro, Portugal, petia@ua.pt
[2] University of Arkansas at Little Rock, USA
[3] Lancaster University, Lancaster, UK

Abstract. EEG source reconstruction is a challenging task and several methods have been applied to this ill-posed inverse problem. Most of the reconstruction techniques rely on imaging models, where the neural activity is described by a dense set of current dipoles. On the other hand, the point source models, which employ a small number of equivalent current dipoles, has received less attention. While both approaches (imaging versus current dipoles) have their own issues, the main advantage of the dipole models is that they approximate summaries of evoked responses or helpful first approximations. In this paper, we use a recursive Bayesian estimation technique, known as Particle Filter (PF), to simultaneously reconstruct the spatial locations within the head and the corresponding waveforms of the most active dipoles that originated the EEG sensor data. Normally, in EEG source reconstruction, fixed dipole locations are assumed. The proposed PF framework presents a shift in the current paradigm by estimating moving EEG sources, which may vary from one location to another in the brain reflecting the underlying brain activity. Our computer simulations, based on generated and real EEG data, show that the proposed PF approach estimates the dynamic EEG sources with high fidelity.

Keywords: brain imaging,particle filters, inverse modeling

1 Introduction

Spatial modeling of EEG sensor data is necessary to make inferences about underlying brain activity. Most source reconstruction techniques belong to one of the two approaches: point source models which explain the data with a small number of equivalent current dipoles or distributed sources known also as imaging models, which use thousands of dipoles. While the imaging approach has been widely studied, the few-dipoles approach has received less attention. The main reason for that is that the inversion of dipole models is harder (nonlinear) problem than the liner inversion of distributed models. However, models with a few dipoles are useful because they represent a direct mapping from scalp topography to a small set of parameters. Dipole models are more suitable for interpretation of observed data, for statistics of dipole parameters across subjects and for engineering solutions as source-based Brain Computer Interfaces (BCI).

The reconstruction of dipole sources from EEG scalp data is not possible without constraints on the inverse solution. As a consequence EEG inverse techniques differ in the assumptions of the constraints. The most popular deterministic inverse methods include the Multiple Signal Classification (MUSIC) algorithm and its modified versions [1], spatial filters (beamformers) based on data-independent [2] or data-driven methods [3] and blind source separation techniques [4]. These approaches, however, are based on the assumption that the source locations are fixed and known *a priori* or perform an exhaustive search of the head volume to find their positions. Given the spatial source locations, they estimate the amplitude and direction of the brain waveforms. Recently, owing to the increase in the available computational power, statistical methods have become feasible within the source localization framework. The Bayesing probability formalism fits well to the inverse problem where the constrains enter as priors and the objective of model inversion is to estimate the conditional or posterior probability of the model parameters. The Variational Bayesian (VB) inversion [5] attempts to estimate both dipole locations and moments. However, VB like all current approaches for EEG source localization, cannot handle the case of moving EEG sources. In this paper, we propose a recursive Bayesian estimation technique, known as Particle Filter (PF), to simultaneously estimate the dipole spatial locations and their corresponding waveforms. This approach presents a shift in the current paradigm by tackling the problem of brain moving source reconstruction.

2 The Particle Filter

Many problems in statistical signal processing, time series analysis and control can be stated in a state-space form. A system transition function describes the prior distribution of a hidden Markov process according to the model

$$\boldsymbol{x}_{k+1} = f_k(\boldsymbol{x}_k, \boldsymbol{w}_k), \tag{1}$$

where f_k is the system transition function and $\boldsymbol{w}_k$ is a zero-mean, white noise sequence of known pdf, independent of past and current states. Measurements $\{\boldsymbol{z}_k; k \in \mathbb{N}\}$ are available at discrete times k. These measurements are related to the state vector via the observation equation

$$\boldsymbol{z}_k = h_k(\boldsymbol{x}_k, \boldsymbol{v}_k), \tag{2}$$

where h_k is the measurement function and $\boldsymbol{v}_k$ is another zero-mean, white noise sequence of known pdf, independent of past and present states and the system noise. Within a Bayesian framework, all relevant information about the state vector $\boldsymbol{x}_k$ given observations up to and including time k can be obtained from the posterior distribution $p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k})$, where $\boldsymbol{z}_{1:k} = \{\boldsymbol{z}_1, \cdots, \boldsymbol{z}_k\}$. This distribution may be obtained recursively in two steps: prediction and update. Suppose that the posterior distribution at the previous time index $k-1$, $p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1})$, is available. Then, using the system transition model, we can obtain the prior pdf of the state at time k

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1}) = \int p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})\ p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1})\ d\boldsymbol{x}_{k-1}. \tag{3}$$

When a measurement at time step k, $\boldsymbol{z}_k$, is available, the prior is updated via the Bayesian rule

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k}) = \frac{p(\boldsymbol{z}_k|\boldsymbol{x}_k)\ p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1})}{p(\boldsymbol{z}_k|\boldsymbol{z}_{1:k-1})}, \tag{4}$$

where the denominator is a normalizing factor, and the conditional pdf of $\boldsymbol{z}_k$ given $\boldsymbol{x}_k$ is defined by the measurement model in (2). The recurrence equations in (3) and (4) constitute the solution to the Bayesian recursive estimation problem. If the functions f_k and h_k are linear and the noises $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are Gaussian with known variances, then an analytic solution to the Bayesian recursive estimation problem is given by the well-known Kalman filter. In the EEG source localization problem, however, the measurement function h_k is non-linear, or, in other words, the EEG measurements $\boldsymbol{z}_k$ are non-linear functions of the source locations $\boldsymbol{x}_k$ (see Section 3). In order to deal with the non-linear and/or non-Gaussian realities, two main approaches have been adopted: parametric and non-parametric. The parametric techniques are based on extensions of the Kalman filter by linearizing non-linear functions around the predicted values. Because of their first-order approximations and unimodal Gaussian assumptions, such extensions found only limited success. The non-parametric techniques are based on Monte Carlo simulations [6]. Specifically, they use a set of random samples, called particles, to estimate the posterior. The posterior is then approximated by a set of weighted particles (hence the name particle filter) as

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k}) \approx \sum_{l=1}^{N} \pi_k^{(l)} \delta(\boldsymbol{x}_k - \boldsymbol{x}_k^{(l)}), \tag{5}$$

where N is the total number of particles and $\pi_k^{(l)} = \frac{w_k^{(l)}}{\sum_{l=1}^{N} w_k^{(l)}}$ is the normalized weight for particle l at time k, and

$$w_k^{(l)} = w_{k-1}^{(l)}\ p(\boldsymbol{z}_k|\boldsymbol{x}_k^{(l)}). \tag{6}$$

Given a discrete approximation to the posterior distribution, one can then proceed to a filtered point estimate such as the mean of the state at time k

$$\hat{\boldsymbol{x}}_k = \sum_{l=1}^{N} \pi_k^{(l)} \boldsymbol{x}_k^{(l)}. \tag{7}$$

The main advantage of the particle filter is that no restrictions are placed on the functions f_k and h_k, or on the distribution of the system and measurement noise. Moreover, the algorithm is quite simple and very easy to implement. Notably, it can be implemented on massively parallel computers, raising the possibility of real time operation with very large sample sets.

3 EEG state space model

Given that the EEG signal is produced by M active dipoles, the relation between the scalp data and primary current density is known as the forward model, [7]

$$\boldsymbol{z}_k = \sum_{m=1}^{M} \boldsymbol{L}_m(\boldsymbol{d}_k(m))\ \boldsymbol{s}_k(m) + \boldsymbol{v}_k \tag{8}$$

where $\boldsymbol{z}_k \in \mathbb{R}^{n_z}$ is the measured multichannel EEG signal from n_z sensors at time k, $\boldsymbol{d}_k(m) \in \mathbb{R}^{3\times 1}$ is the 3D brain source location of the m^{th} dipole at time k,

$\boldsymbol{L}_m(\boldsymbol{d}_k(m)) \in \mathbb{R}^{n_z\times 3}$ is the lead field matrix for dipole m, $\boldsymbol{s}_k(m) \in \mathbb{R}^{3\times 1}$ is the source signal of the m^{th} dipole at time k (termed also dipole moment) and $\boldsymbol{v}_k$ is a white Gaussian process noise with variance $\sigma_v^2\boldsymbol{I}$. The entries of the lead field matrix $\boldsymbol{L}_m$ are non-linear functions of the dipole location, electrodes' positions and head geometry . Observe also, from Eq. (8) that the EEG measurements $\boldsymbol{z}_k$ are linear with respect to the dipole moments $\boldsymbol{s}_k$, but non-linear with respect to the source locations $\boldsymbol{d}_k$. This model is routinely used in most clinical and research applications to EEG source localization. From Eq. (8), we can compute the likelihood of each measurement as

$$\mathcal{L}(\boldsymbol{z}_k|\boldsymbol{x}_k) \propto \exp\left[-\frac{(\boldsymbol{z}_k - \boldsymbol{L}(\boldsymbol{d}_k)\boldsymbol{s}_k)^t \boldsymbol{R}_{z_k}^{-1}(\boldsymbol{z}_k - \boldsymbol{L}(\boldsymbol{d}_k)\boldsymbol{s}_k)}{2}\right], \tag{9}$$

where $\boldsymbol{R}_{z_k}$ is the covariance matrix of the measurement vector $\boldsymbol{z}_k$, $\propto$ denotes "proportional to". $\boldsymbol{d}_k = [\boldsymbol{d}_k(1)^t, \cdots, \boldsymbol{d}_k(M)^t]^t \in \mathbb{R}^{3M\times 1}$ represent the spatial coordinates of M dipoles within the head at time k,

$\boldsymbol{L}(\boldsymbol{d}_k) = [\boldsymbol{L}_1(\boldsymbol{d}_k(1))^t, \cdots, \boldsymbol{L}_M(\boldsymbol{d}_k(M))^t]^t \in \mathbb{R}^{n_z\times 3M}$ is the lead field matrix of M dipoles at time k, and $\boldsymbol{s}_k = [\boldsymbol{s}_k(1)^t, \cdots, \boldsymbol{s}_k(M)^t]^t \in \mathbb{R}^{3M\times 1}$ is the 3D propagation of brain source signals for M dipoles. The state vector, at iteration k, $\boldsymbol{x}_k = [\boldsymbol{d}_k^t, \boldsymbol{s}_k^t]^t$, contains the geometrical positions of the M dipoles and their corresponding signals. In the most general case, having no any prior knowledge of the source locations or signals, we choose to model the state transition as a random walk (first-order Markov chain) in the source localization space

$$\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \boldsymbol{w}_k, \tag{10}$$

where $\boldsymbol{w}_k$ is a zero-mean, Gaussian white noise sequence with covariance $\sigma_w^2\ \boldsymbol{I}$, where $\boldsymbol{I}$ denotes the identity matrix. The process $\boldsymbol{w}_k$ is assumed to be independent of past and current states. Within the particle filtering framework, the state-space model of the dipole source localization problem is then given by:

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \boldsymbol{w}_k : & \text{state transition model,} \\ \boldsymbol{z}_k = \boldsymbol{L}(\boldsymbol{x}_k)\ \boldsymbol{s}_k + \boldsymbol{v}_k : & \text{observation model.} \end{cases} \tag{11}$$

4 Results on simulated EEG data

In order to assess the performance of the proposed PF algorithm, we first generate various EEG data sets (the ground truth) by randomly choosing the (x, y, z) coordinates of two, three and four fixed and moving dipoles from a grid of 21012 uniformly distributed dipoles. We assume that the state-space is discrete and consists of a finite number of states. The coordinates of a grid of dipoles ($\boldsymbol{D}$) normally distributed inside a spherical head model, with radius $R = 10$ cm, are defined as $\boldsymbol{D} = \{d_{ij} = [x_{ij}, y_{ij}, z_{ij}]\}$. For the moving dipoles, the initial and the final locations are selected. It is assumed that the locations are fixed over a duration of 40 ms, which correspond to 20 samples. We simulated four sequential location stages across 80 samples. The dipoles move linearly between the initial and final locations with constant speed. Sinusoidal waveforms with amplitudes 0.1 and frequencies $[10, 15, 20, 25]$ Hz are assumed to be the brain signals originated from the dipoles. For the moving dipoles, the waves propagate along the moving directions of each dipole. We can then simulate the EEG signal with varying SNR using the forward model in Eq. (8). Each dipole d_i is associated with 6 variables in the state vector, three space coordinates $\{x_{ij}, y_{ij}, z_{ij}\}$ and one amplitude in each direction $\{s_{xi}, s_{yi}, s_{zi}\}$. For the initial state vector, $N = 500$ samples were randomly generated from a normal distribution in the interval $\boldsymbol{x}_0 \in [\min(\boldsymbol{D}), \max(\boldsymbol{D})]$. Table 1 summarizes the Mean Square Distance Error (MSDE) for the estimation of two, three and four fixed and moving dipoles with varying SNR (0.5 dB, 5 dB, 10 dB), where MSDE is

$$\text{MSDE} = \sqrt{(\hat{x} - x)^2 + (\hat{y} - y)^2 + (\hat{z} - z)^2} \qquad (12)$$

Observe that the proposed PF algorithm appears to be especially suitable for estimation of moving dipoles. Taking into account the fact that dynamic EEG sources is a more plausible assumption, from a biological perspective, than fixed EEG sources, the PF framework presents a shift in the current approach of fixed EEG source localization. This is especially important for laying down a theoretical basis for source-based BCI technology, which relies on an accurate estimation of the dynamic EEG brain sources.

Table 1. MSDE $[mm]$ of fixed and moving dipole locations

Number of fixed dipoles	SNR = 10dB	SNR = 5dB	SNR = 0.5dB
2	5.9×10^{-3}	6.3×10^{-3}	7.61×10^{-2}
3	7.1×10^{-3}	8.17×10^{-3}	9.33×10^{-2}
4	6.32×10^{-3}	7.1×10^{-3}	6.50×10^{-2}
Number of moving dipoles	**SNR = 10dB**	**SNR = 5dB**	**SNR = 0.5dB**
2	0.8×10^{-4}	0.3×10^{-3}	2.19×10^{-3}
3	1.7×10^{-4}	0.8×10^{-3}	3.9×10^{-3}
4	1.5×10^{-4}	0.6×10^{-3}	3.5×10^{-3}

5 Results on real EEG data

In this section, the PF is validated with real EEG data. Visually Evoked Potentials (VEPs) were recorded from 13 subjects. Images of transparently superimposed human faces and houses were presented as visual stimuli in a sequence of 300 ms each and were preceded by a fixation cross displayed for 500 ms. The inter-trial interval was 2000 ms. The participants had to decide if the face or the house is the same as the one presented on the previous trial. EEG signals were recorded from 20 electrodes (Fp1, Fp2, F3, F4, C3, C4, P3, P4, O1, O2; F7, F8, T3, T6; P7, P8, Fz, Cz, Pz, Oz) according to the 10/20 International system. VEPs were calculated off-line averaging segments of 400 points of digitized EEG (12 bit A/D converter, sampling rate 250 Hz). These segments covered 1600 ms comprising a pre-stimulus interval of 148 ms (37 samples) and post-stimulus onset interval of 1452 ms. The experimental setup was designed by Santos et al. [8] for their study on subject attention and perception using VEP signals.

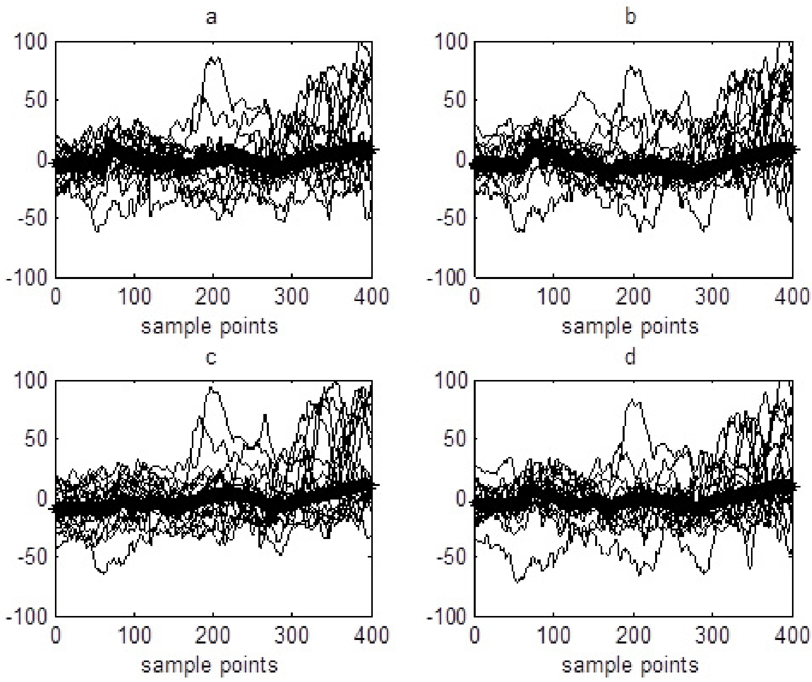

Fig. 1. Superposition of 18 VEP trials measured at four electrodes ($O1, O2, Pz, Oz$). The bold trace represents the average of all trials used to test the fPF and the PF & BF approaches.

Fig. 1 represents 18 enhanced (by Principal Component Analysis) trials of four channels. In the reconstructed signals, it is possible to identify a positive peak in the range of 80-120 milliseconds, known as P100. P100 corresponds to the perception of the sensory stimulus, a brain activity that is known to happen in the primary visual cortex. Therefore, the occipital channels (O1, 02, Oz) and the parietal channels (Pz, P3, P4) located around the visual cortex are expected to respond with stronger VEPs.For illustration purposes we choose to reconstruct only the two most active dipoles.

Comparison with the Beamforming-based PF. The Beamforming (BF) approach, originated in radar and sonar field, is a popular approach for estimating EEG source signals [9]. The BF assumes that the brain sources number and locations are known *a priori* and estimates the dipole moments using a spatial filter. The BF spatial filter consists of weight coefficients that, when multiplied by the electrode measurements, give an estimate of the dipole moment at time k, i.e., $\boldsymbol{s}_k = \boldsymbol{W}^t \boldsymbol{z}_k$, where $\boldsymbol{W} \in \mathbb{R}^{n_z \times 3M}$ is the weighting matrix. The choice of the BF weights is based on the statistics of the EEG signal received at the electrodes. The objective is to optimize the BF response with respect to a prescribed criterion, so that the output $\boldsymbol{s}_k$ contains minimal contribution from noise and interference. There are a number of criteria for choosing the optimum weights. We propose to minimize the output signal variance in order to provide good signal estimation. Minimization of contributions to the output due to interference is accomplished by choosing the weights to minimize the variance of the filter output. Thus we have $var[\boldsymbol{s}_k] = Tr[\boldsymbol{W}^t \boldsymbol{R}_{z_k} \boldsymbol{W}]$, where $Tr[\bullet]$ denotes the trace of the matrix in brackets. To ensure that the desired signal is passed with unity gain, a constraint may be used so that the BF response to the desired signal satisfies $\boldsymbol{W}^t \boldsymbol{L}(\boldsymbol{x}_k) = \boldsymbol{I}$. The constrained optimization problem can then be expressed $\boldsymbol{W}^* = \text{argmin}_{\boldsymbol{W}}\ Tr[\boldsymbol{W}^t \boldsymbol{R}_{z_k} \boldsymbol{W}]$ subject to $\boldsymbol{W}^t \boldsymbol{L}(\boldsymbol{x}_k) = \boldsymbol{I}$. The optimal solution can be derived by constrained minimization using Lagrange multipliers $\boldsymbol{W}^* = \boldsymbol{R}_{z_k}^{-1} \boldsymbol{L}(\boldsymbol{x}_k)[\boldsymbol{L}(\boldsymbol{x}_k)^t \boldsymbol{R}_{z_k}^{-1} \boldsymbol{L}(\boldsymbol{x}_k)]^{-1}$. This approach is often called the linearly constrained minimum variance (LCMV) beamformer.

We compare the proposed fully Bayesian PF framework with the beamformer-based PF to estimate the two strongest sources (d_1 and d_2) that may have produced the P100 peak. The results show that in both approaches the final coordinates correspond to the zone of the primary visual cortex as illustrated in Fig. 2. In this sense, both approaches seem to be coherent in identifying the two strongest brain sources, producing the P100 peak. However, the more symmetric coordinates estimated by the PF d_1 (3.6 mm,-5.5 mm, -1.03 mm) and d_2 (-3.3 mm, -2.2 mm, -1.02 mm) are more biologically plausible and easy to interpret than the beamformer-based PF with d_1 (0.71 mm, -6.3 mm, -1.9 mm) and d_2 (6.8 mm, -2 mm, -6.14 mm).

6 Conclusions

In this paper, we developed a particle filter (PF) framework to simultaneously reconstruct the spatial coordinates and waveforms of moving brain dipoles. Our main contribution is the assumption that the dipoles are dynamic which is biologically plausible.

We conducted extensive simulations, based on generated and real EEG experiments, in order to study the accuracy and robustness of the proposed algorithm.

Our simulations show that the PF converges to the correct dipole coordinates as long as the power of the signal is higher than the power of the noise within the EEG measurements. We have also conducted EEG experiments, where subjects were exposed to various visual stimuli. The PF pointed out correctly dipoles inside the visual cortex zone as the ones that most probably have produced the observed EEG signal. The PF approach reveals to be less dependent on *a priori* knowledge about the brain sources. Even when the initial guess is completely erroneous, the tracker still moves to the active brain zones.

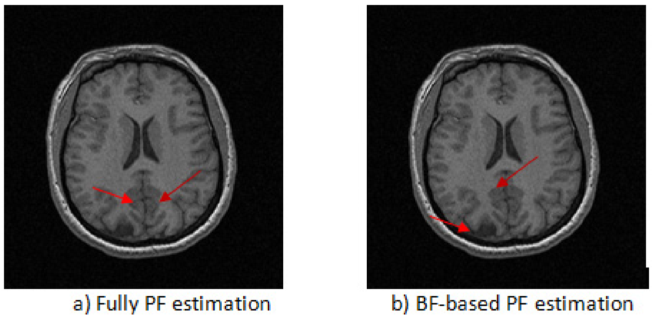

Fig. 2. Axial view of primary visual cortex zone. The arrows point the estimated source locations.

Acknowledgements. This work was funded by the Portuguese Foundation for Science and Technology (FCT) in the framework of the CMU-Portugal Faculty Exchange program 2012.

References

1. S. Baillet, J.C. Mosher, R.M. Leahy: Electromagnetic brain mapping IEEE Signal Processing Magazine, 18(6), 14–30, (2001)
2. J. Gross, A.A. Ioannides: Linear transformations of data space in MEG Physics in Medicine and Biology, 44(8), 2081–2097, (1999)
3. B.D. Van Veen, W. Van Drongelen, M. Yuchtman, A. Suzuki: Localization of Brain Electrical Activity via Linearly Constrained Minimum Variance Spatial Filter: IEEE Transactions on Biomedical Engineering, 44(9), 867–880, (1997)
4. A. Cichocki, S.I. Amari: Adaptive Blind Signal and Image Processing: Learning Algorithms and Applicaions John Wiley & Sons, (2002)
5. Kiebel S.J., Daunizeau J.Phillips C., Friston K.J.: Variational Bayesian inversion of the equivalent current dipole model in EEG/MEG NeuroImage, 39(2),728–741 (2008)
6. Doucet A, Godsill S, Andrieu C On sequential Monte Carlo sampling methods for Bayesian filtering. Statistics and Computing, 10 197–208, (2000)
7. S. Sanei, J.A. Chambers: EEG Signal Processing John Wiley & Sons, (2007)
8. Santos I.M., J. Iglesias, E. I. Olivares, A.W. Young: Differential effects of object-based attention on evoked potentials to fearful and disgusted faces Neuropsychologia, 46(5) 1468–1479, (2008)
9. Georgieva P, Mihaylova L, Bouaynaya N, Jain L : Particle filters and beamforming for EEG source estimation IEEE World Congress on Computational Intelligence, International Joint Conference on Neural Networks. Brisbane, (2012)