Gas Permeation Homework
Rowan University
Rate Controlled Separations

Perfect Mixing Case:

$$x_{RA} = \frac{0.21 - y_{PA}\theta}{1-\theta} \qquad \text{Eq (1)}$$

$$\alpha_{AB} = \frac{y_{PA}/x_{RA}}{(1-y_{PA})/(1-x_{RA})} \qquad \text{Eq (2)}$$

$$\alpha_{AB} = \alpha^*_{AB}\left[\frac{x_{RA}(\alpha_{AB}-1)+1-r\alpha}{x_{RA}(\alpha_{AB}-1)+1-r}\right] \qquad \text{Eq (3)}$$

1. Specify values of θ between zero and one. This leaves three unknowns (x_{RA}, y_{PA} and α_{AB}) that must be calculated for each value of θ.
2. Solve the equations simultaneously for the unknowns. This can be done using Excel Solver or a similar package. The general procedure is :
   a. choose a value of y_{PA},
   b. calculate x_{RA} using the first equation, and then calculate x_{RA} using Eq (1)
   c. calculate α using Eq (2) and redo using Eq (3).
   d. compare the two calculated results for α. If the values are not equal, go back to step (a) and choose a different value of y_{PA}
3. Area can be calculated using the equation

$$A = y\frac{y_{PA}n_P}{\overline{P}_{MA}(x_{RA}P_R - y_{PA}P_P)}$$

Results

θ	**x_{RA}**	**y_{PA}**	**α**	**A (ft^2)**
0.01	0.208	0.406	2.602	22,000
0.2	0.174	0.353	2.587	462,000
0.4	0.146	0.306	2.574	961.000
0.6	0.124	0.267	2.563	1,488,000
0.8	0.108	0.236	2.555	2,035,000
0.99	0.095	0.211	2.548	2,567,000

The separation factor remains almost constant. The maximum oxygen concentration in the permeate occurs with the smallest permeate rate (θ=0.01), which is simply a trade-off between quality and quantity.

At θ=0.4, only 60% of the feed is retained… but the retentate concentration has increased only a little bit from 79 to 85.4 mol%. Membrane area is large. Need a membrane with better selectivity, a module with better flow characteristics, or two modules in series.

Cross-flow configuration:

$$y_{PA} = x_{RA}^{\left(\frac{1}{1-\alpha}\right)}\left(\frac{1-\theta}{\theta}\right)\left[1 - x_{RA}^{\left(\frac{\alpha}{\alpha-1}\right)}\left(\frac{x_{FA}}{1-x_{FA}}\right)^{\left(\frac{\alpha}{\alpha-1}\right)} - x_{RA}^{\left(\frac{\alpha}{\alpha-1}\right)}\right]$$

$$x_{FA}n_F = x_{RA}(1-\theta)n_F + y_{PA}\theta n_F \qquad or \qquad xRA = \frac{(x_{FA} - y_{PA}\theta)}{1-\theta}$$

Solution Procedure:

Solve the 2 equations simultaneously for each value of θ between 0 and 1.0. This can be done automatically using Excel Solver. The general procedure is:

1. Guess a value of y_{PA} between 0 and 1.
2. Calculate x_{RA} using the second equation above.
3. Calculate y_{PA} using the first equation above. You will have to plug in the calculated value of x_{RA} from step (2).
4. Compare the calculated value of y_{PA} from step (3) to the guessed value in step (1). If they are not the same, go back to step (1) and change the value of your guess for y_{PA}. Repeat until the values agree.
5. Calculate the stage separation factor:

$$\alpha_s = \frac{y_{PA}/x_{RA}}{(1-y_{PA})/(1-x_{RA})}$$

The results are:

θ	**x_{RA}**	**y_{PA}**	**α**
0.01	0.208	0.407	2.61
0.2	0.168	0.378	3.01
0.4	0.122	0.342	3.74
0.6	0.0733	0.301	5.44
0.8	0.0274	0.256	12.2
0.99	0.000241	0.212	1120

Comments:
For crossflow, the permeate is richer in oxygen and the retentate is richer in nitrogen – compared with the results for the ideally mixed module. More efficient separation.

For the crossflow case, the degree of separation based on the mole fractions in the permeate and the retentate exiting the stage (α_s), increases with increasing cut. For values of cut > 0.2, this value is greater than the ideal separation factor α^*!

For the ideally mixed case, α_s decreases slightly with cut from 2.603 (θ=0.01) to 2.548 (θ=0.99). Thus the separation factor is relatively constant for the ideally mixed case. The value of α_s is always less than the ideal separation factor α^* for this case.