Week 5 Lecture

Viscosity, viscoelasticity, gelation

Viscosity

Demo: Ice melts to become water--Water flows

Demo: A cube of gelatin gel deforms under shear



What happens if you warm the gelcan you still push it?

Liquid cannot sustain a shear—it flows when sheared

Resistance to flow depends on viscosity

Viscosity of oil

Oil is more viscous than water

Demo: Takes a longer time to flow through funnel

Demo—you did this in the lab:
Drop a ball bearing or chickpea in a tube
filled with water, oil, and honey
Which liquid does the ball fall slowest, fastest?
Rank the time of fall in increasing order

Viscosity

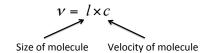
A material is a liquid if the molecules can move around each other.

The fundamental quantity that governs this is the *time* that it takes for molecules to move around their neighbors.

If it takes a long time for them to move by each other, the material is very viscous.

If it takes a short time, the material is less viscous.

Molecular viscosity



Molecular viscosity

Scientists typically use two measures of viscosity:

 $v = l \times c$

kinematic viscosity

 $\eta = \rho v$

Dynamic viscosity

For lay people, the best measure of viscosity is the time it takes something to flow.

Of course, these three concepts (kinematic and dynamic viscosity, and flow time) are related.

Molecular viscosity

length²/time

$$l = l \times c$$
 length²/time

molecular viscosity: $v = l \times c$

Recall: elasticity: $E = \frac{k_B T}{I^3}$

Origin of viscosity of water

Dimensional analysis gives a relation between E and $\boldsymbol{\eta}$

$$\eta = E \times \tau$$

Viscosity = Elasticity times a relaxation time

All fluids are solids at short enough times

Times depend on the fluid

Short-time elasticity of a liquid



http://www.youtube.com/watch?v=f2XQ97XHjVw

Origin of viscosity of water

$$\eta = E \times \tau$$
 Dynamic viscosity

$$\eta =
ho imes
u$$
 kinematic viscosity

Just different measures of viscosity

Relaxation time for water

Question: What is the time that molecules move by each other?

Answer:

$$\tau = \frac{l}{c}$$

l = molecular size = 5 Angstroms
c = 1500 meters/second (room Temperature)

$$\tau = \frac{5 \times 10^{-8} \, \text{cm}}{150000 \, \text{cm/sec}} \approx 3 \times 10^{-13} \text{sec}$$

Relaxation time for water

Compare this to

$$\tau = \frac{\rho v}{E}$$

 $E = kT/I^3 = 2.5 \times 10^{10} g / (cm. sec^2)$

 ρ = 1 g/cm³

 $v = 0.01 \text{ cm}^2/\text{sec}$

$$\tau = \frac{0.01}{2.5 \times 10^{10}} \approx 4 \times 10^{-13} \text{sec}$$

Viscosity of hot oil

Hot oil flows faster than cold oil Viscosity decreases with increasing temperature Molecules move around each other more easily

$$\eta = E \times \tau$$

$$\tau = \tau_0 \exp(U/kT)$$

$$\eta = E \times \tau_0 \exp(U/kT)$$

U: same interaction energy as in week 1



Thickeners—viscosity depends on concentration

- Very small amount of material increases viscosity a lot example adding flour to a cream sauce or to a gravy
 - Related to thickeners being polymers and gelation



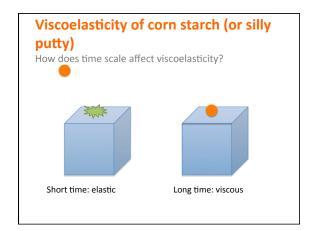
Polymers and Gelation

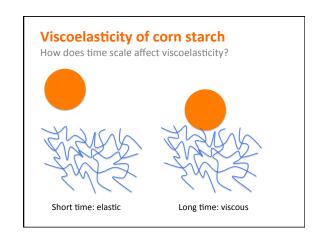
Equations:
$$E = \frac{k_B T}{t^3}$$
 $\eta = E \times \tau$

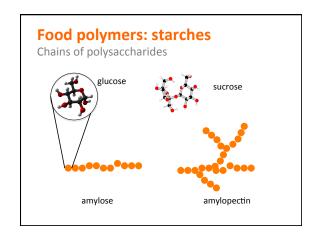
Viscoelasticity and time-dependence Elasticity of polymer gels

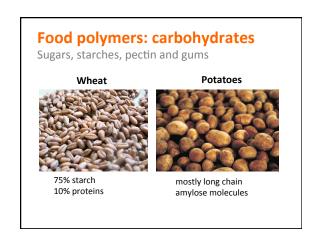
Viscosity of polymer solutions

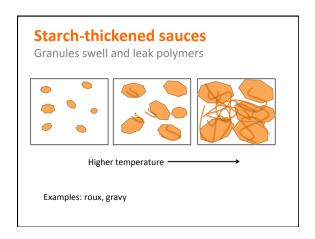


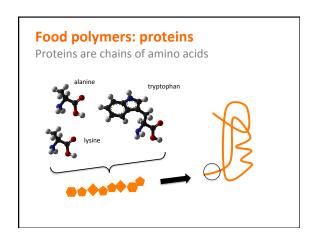


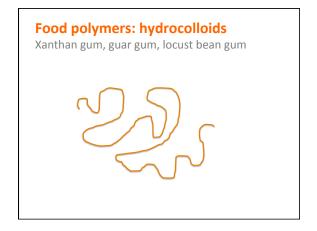


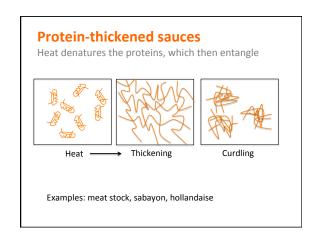


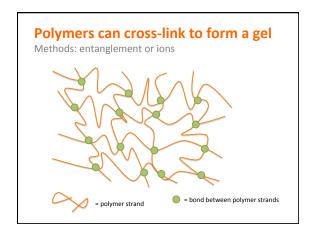


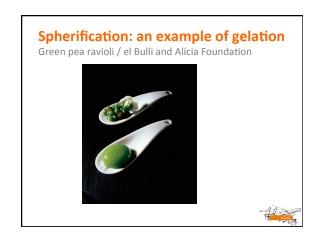




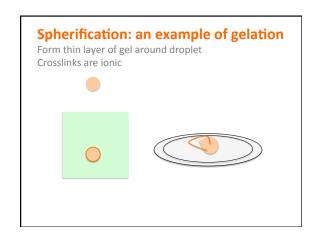


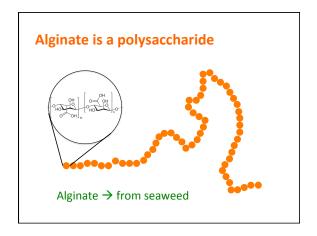


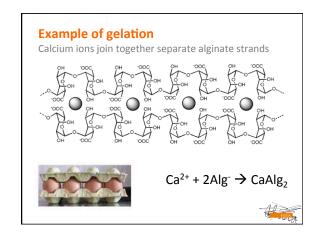


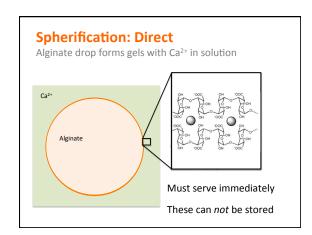


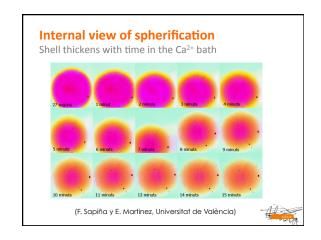


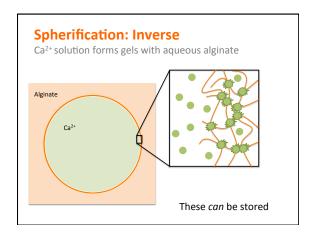


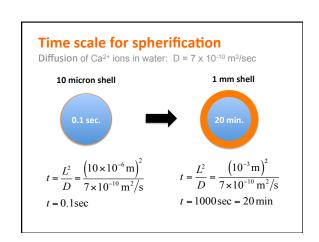


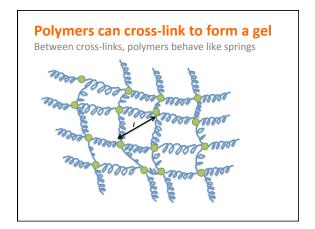


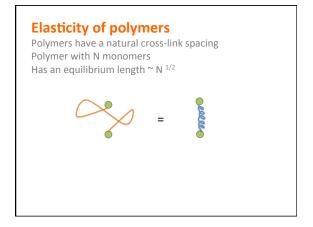


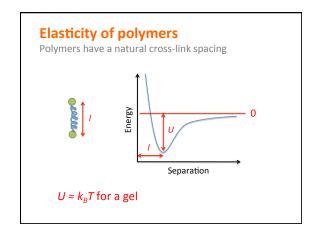


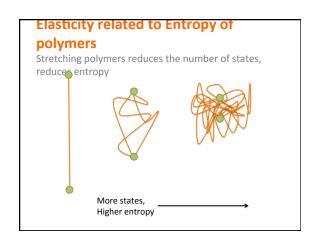


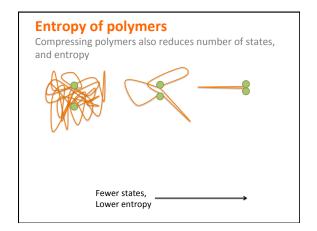


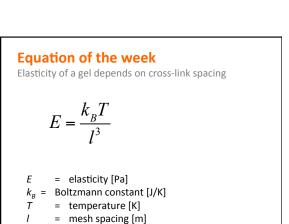












Equation of the week

Elasticity of a gel depends on cross-link spacing

$$E = \frac{k_B T}{l^3} \qquad [E] = \frac{J}{m^3}$$

$$[E] = \frac{J}{m^3}$$

= elasticity [Pa]

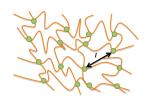
 k_B = Boltzmann constant [J/K]

= temperature [K]

= mesh spacing [m]

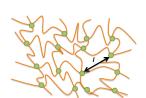
Mesh size

Calculate the spacing between cross-links



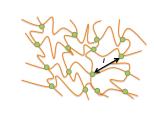
Mesh size

Calculate the spacing between cross-links



Mesh size

Calculate the spacing between cross-links



 $4.2 \times 10^{-21} J$ 20kPa $= 2.1 \times 10^{-25} \text{m}^3$

 $l = \sqrt[3]{2.1 \times 10^{-25} \,\mathrm{m}^3}$

 $=6 \times 10^{-9} \text{ m}$

= 6 nm

Melting of gels

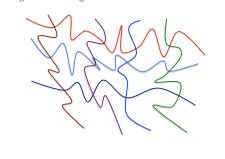
The cross-links in gelatin detach at high temperatures

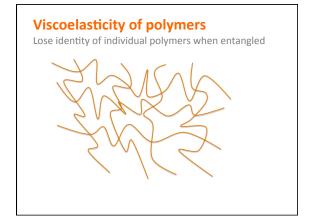


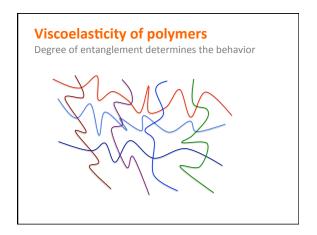
Higher temperature -

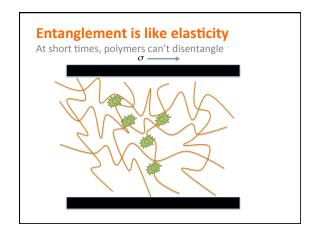
Viscoelasticity of polymers

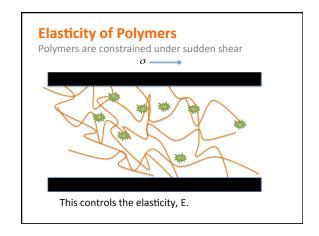
Degree of entanglement determines the behavior

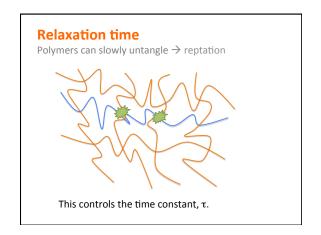


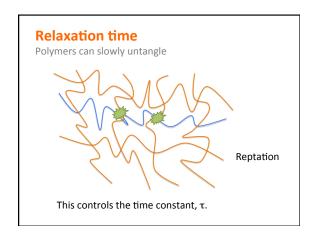












Equation of the week

Viscosity is related to elasticity by a time-scale

$$\eta = E \times \tau$$

 η = viscosity [Pa s]

E = elasticity [Pa]

 τ = time-constant [s]

Concentration Dependence

Polymers become entangled at high densities



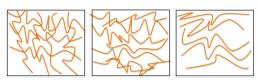
Higher concentration

This sets the elasticity, E, and the relaxation time, τ .

Shorter polymers don't entangle as much \rightarrow lower η

Shear-rate Dependence

Polymers can disentangle at high shear rates

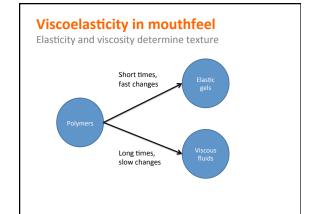


Animation courtesy Naveen Sinha

Higher shear -

Shear thinning

Example: ketchup



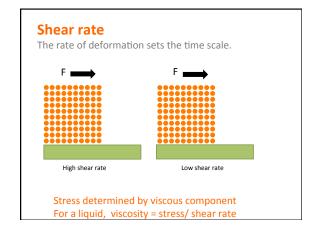
Equation of the week

Sample calculation for honey

$$\eta = E \times \tau$$

$$\tau = \frac{\eta}{E} = \frac{10^3 \,\mathrm{Pa} \cdot \mathrm{s}}{2 \times 10^3 \,\mathrm{Pa}}$$

$$\tau = 0.5$$
s

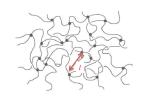


Solid gel



Thickener is a polymer Polymer forms network in the water This forms the solid gel

Solid gel



Thickener is a polymer Polymer forms network in the water This forms the solid gel

$$e^{33} = \frac{kT}{E}$$

$$= \frac{4.2 \times 10^{-21}}{1000}$$

$$= 4.2 \times 10^{-24}$$

 $l = \sqrt[3]{4.2 \times 10^{-24}}$

= 1.6×10^{-8} meters

= 16 nanometers

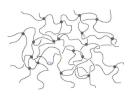
Viscosity of thickeners



The bonds are not permanent Molecules can move Molecules must disentangle to move

Spaghetti demo

Viscosity of thickeners

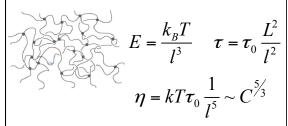


$$\eta = E \times \tau$$

$$E = \frac{k_B T}{I^3} \qquad \tau = \tau_0 \frac{L^2}{I^2}$$

Viscosity of thickeners

$$\eta = E \times \tau$$



Very strong concentration dependence