# Translating Euler's Works:
## How to get your students' feet wet in undergraduate research

"*It appears to me that if one wants to make progress in mathematics one should study the masters, and not the pupils*." ---- Niels Abel's

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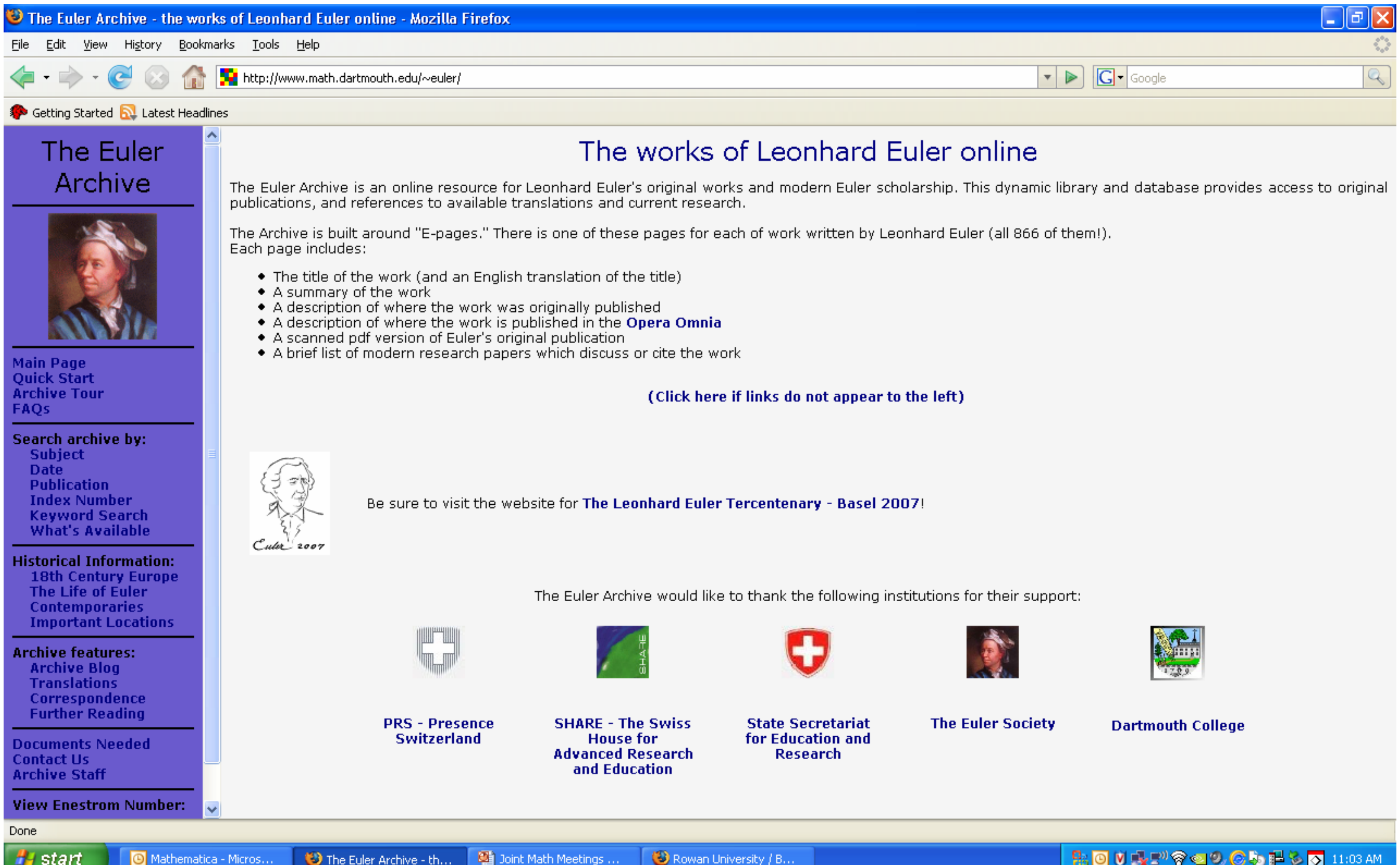
Joint Math Meetings
San Diego, CA
January 6, 2008

# Euler’s Works Online

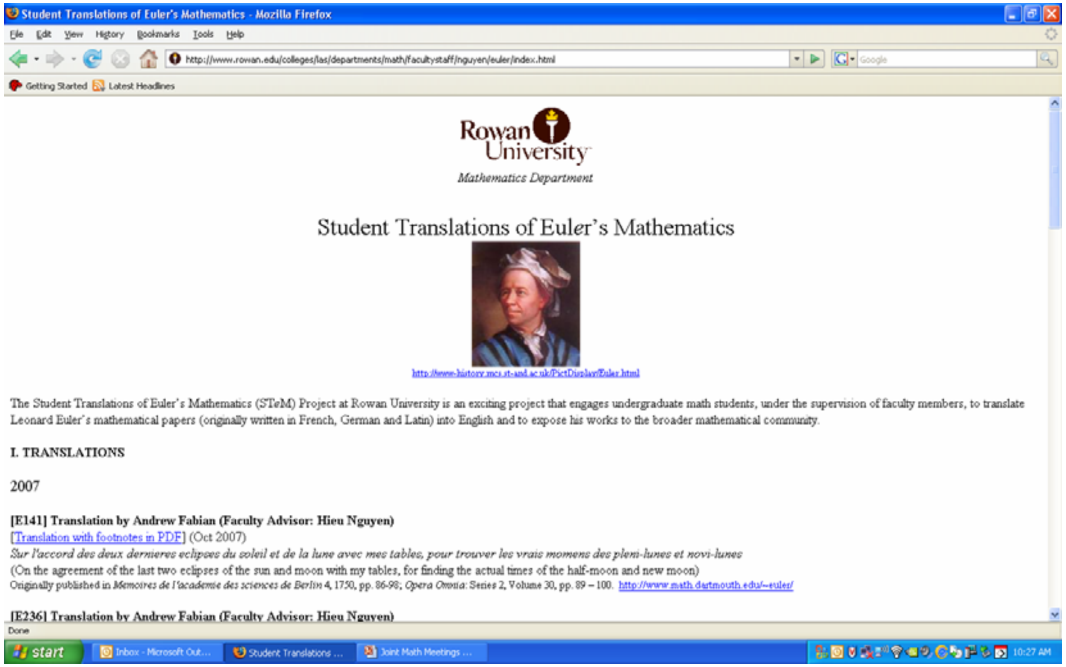
The Euler Archive: (Domini Klyve and Lee Stemkoski)

http://www.math.dartmouth.edu/~euler/



# Student Translations of Euler’s Mathematics (STEM):

http://www.rowan.edu/colleges/las/departments/math/facultystaff/nguyen/euler/index.html




- Started by Thomas Osler and his students in 2006
- Eight Rowan students and three faculty members involved to date
- Nine translations completed by six students; four translations with synopsis posted on The Euler Archive

# Recent Translations

## Andrew Fabian

Four translations (two on astronomy; two on analysis)

**[E141] Translation by Andrew Fabian** (Oct 2007)
*Sur l'accord des deux dernieres eclipses du soleil et de la lune avec mes tables, pour trouver les vrais momens des pleni-lunes et novi-lunes* (On the agreement of the last two eclipses of the sun and moon with my tables, for finding the actual times of the half-moon and new moon) Originally published in *Memoires de l'academie des sciences de Berlin* 4, 1750, pp. 86-98; *Opera Omnia*: Series 2, Volume 30, pp. 89 – 100.

**[E236] Translation by Andrew Fabian** (Aug 2007)
*Exposition de quelques paradoxes dans le calcul integral* (Explanation of Certain Paradoxes in Integral Calculus) Originally published in *Memoires de l'academie des sciences de Berlin* 12, 1758, pp. 300-321; *Opera Omnia*: Series 1, Volume 22, pp. 214 – 236.

# Excerpt from E236



EXPOSITION DE QUELQUES PARADOXES
DANS LE CALCUL INTÉGRAL

PAR M. EULER.

*Premier Paradoxe.*

I.

Je me propoſe ici de développer un paradoxe dans le calcul intégral, qui paroitra bien étrange : c'eſt qu'on parvient quelquefois à des équations différentielles, dont il paroit fort difficile de trouver les intégrales par les régles du calcul intégral, & qu'il eſt pourtant aiſé de trouver, non par le moyen de l'intégration, mais plutôt en différentiant encore l'équation propoſée ; de ſorte qu'une différentiation réitérée nous conduiſe dans ces cas à l'intégrale cherchée. C'eſt ſans doute un accident fort ſurprenant, que la différentiation nous puiſſe mener au même but, auquel on eſt accoutumé de parvenir par l'intégration qui eſt une opération entierement oppoſée.

II. Pour mieux faire ſentir l'importance de ce paradoxe, on n'a qu'à ſe ſouvenir, que le calcul intégral renferme la méthode naturelle de trouver les intégrales des quantités différentielles quelconques : & de là il ſemble qu'une équation différentielle étant propoſée, il n'y a d'autre moyen pour arriver à ſon intégrale, que d'en entreprendre l'intégration. Et ſi l'on vouloit, au lieu d'intégrer cette équation, la différentier encore une fois, on devroit croire qu'on s'éloigneroit encore davantage du but propoſé ; attendu qu'on auroit alors une équation différentielle du ſecond degré, qu'il faudroit même deux fois intégrer, avant qu'on parvint au but propoſé.

III.

# EXPLANATION OF CERTAIN PARADOXES IN INTEGRAL CALCULUS

*BY MR. EULER*
*Translation from the French: ANDREW FABIAN*

*The First Paradox*

I.

Here I intend to explain a paradox in integral calculus that will seem rather strange: this is that we sometimes encounter differential equations in which it would seem very difficult to find the integrals by the rules of integral calculus yet are still easily found, not by the method of integration, but rather in differentiating the proposed equation again; so in these cases, a repeated differentiation leads us to the sought integral. This is undoubtedly a very surprising accident, that differentiation can lead us to the same goal, to which we are accustomed to find by integration, which is an entirely opposite operation.

II. To get a better feel for the importance of this paradox, we only have to remember that integral calculus holds the natural method for finding integrals from differential quantities: and from this it seems that for a proposed differential equation, there is no other way to arrive at its integral than to attempt its integration. And if we would, instead of integrating this equation, differentiate it once more, we would need to believe that we would further distance ourselves from the proposed goal; considering that we would then have a differential equation of the second degree, it would need two integrations before we reach the proposed goal.

III. Il doit donc être très ſurprenant, qu'une différentiation réiterée ne nous éloigne non ſeulement davantage de l'intégrale, que nous nous propoſons de chercher, mais qu'elle nous puiſſe même fournir cette intégrale. Ce ſeroit ſans doute un grand avantage, ſi cet accident étoit général, & qu'il eut lieu toujours, puisqu'alors la recherche des intégrales, qui eſt ſouvent même impoſſible, n'auroit plus la moindre difficulté : mais il ne ſe trouve qu'en quelques cas très particuliers dont je rapporterai quelques exemples : les autres cas demandent toujours la méthode ordinaire d'intégration. Voilà donc quelques problèmes qui ſerviront à éclaircir ce paradoxe.

## PROBLEME I.

*Le point* A *étant donné, trouver la courbe* EM *telle, que la perpendiculaire* AV *tirée du point* A *ſur une tangente quelconque de la courbe* MV, *ſoit partout de la même grandeur.* Fig. 1.

IV. Prenant pour axe une droite quelconque AP, tirée du point donné A, qu'on y tire d'un point quelconque de la courbe cherchée M la perpendiculaire MP, & une autre infiniment proche $mp$ : & qu'on nomme $AP = x$, $PM = y$, & la longueur donnée de la ligne $AV = a$. Soit de plus l'élément de la courbe $Mm = ds$, & ayant tiré $M\pi$ parallele à l'axe AP, on aura $Pp = M\pi = dx$ & $\pi m = dy$ ; donc $ds = \sqrt{(dx^2 + dy^2)}$. Qu'on baiſſe du point P auſſi ſur la tangente MV la perpendiculaire PS, & ſur celle-cy du point A la perpendiculaire AR, qui ſera parallele à la tangente MV. Maintenant, puisque les triangles PMS & APR ſont ſemblables au triangle $Mm\pi$, on en tirera : $PS = \frac{M\pi \,.\, PM}{Mm} = \frac{y\,dx}{ds}$ & $PR = \frac{m\pi \,.\, AP}{Mm} = \frac{x\,dy}{ds}$ : d'où, à cauſe de $AV = PS - PR$, nous aurons cette équation, $a = \frac{y\,dx - x\,dy}{ds}$ ou $y\,dx - x\,dy = a\,ds$

Pp 3 $\qquad = a$

III. It must therefore be very surprising that a repeated differentiation does not distance us only further from the integral that we proposed to find, but it can even give us this integral. This would undoubtedly be a great advantage, if this accident were general and always held true, since then the study of integrals, which are often impossible, would no longer pose the least difficulty: but it is only found in some very particular cases in which I will relate some examples: the other cases always follow the ordinary method of integration. Therefore, here are some problems that serve to clarify this paradox.

## PROBLEM I

*Given point A, find the curve EM such that the perpendicular AV, derived from point A onto some tangent of the curve MV, is the same size everywhere. (Fig. 1)*

IV. Taking for the axis some straight line $AP$ derived from the given point $A$, we derive the perpendicular $MP$ there from some point $M$ on the sought curve and another infinitely close line $mp$. Also, let us call $AP = x$, $PM = y$, and the given length of the line $AV = a$. Furthermore, let the element of the curve $Mm = ds$, and having derived $M\pi$ parallel to the axis $AP$, we will have $Pp = M\pi = dx$ and $\pi m = dy$; therefore $ds = \sqrt{(dx^2 + dy^2)}$. We extend from the point $P$ also onto the tangent $MV$ the perpendicular $PS$ and onto this line from the point $A$ the perpendicular $AR$, which will be parallel to the tangent $MV$. Now, since the triangles $PMS$ and $APR$ are similar to the triangle $Mm\pi$, we can derive: $PS = \frac{M\pi \cdot PM}{Mm} = \frac{ydx}{ds}$ and $PR = \frac{m\pi \cdot AP}{Mm} = \frac{xdy}{ds}$: from

# Elizabeth Volz

Partial translation of a series of seven Euler-Goldbach letters (in German) dated between Nov 1741 and June 1742 discussing Leuneschlos' Paradox:

$$\frac{2^{i}+2^{-i}}{2} \approx 10/13$$

$$0.769239 \approx 0.769231$$

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Johannes Leuneschlos (Portrait[1])
Professor of Mathematics (1650-1700)
University of Heidelberg

# Kathryn Robertson

**[E797] Translation by Kathryn Robertson** (Dec 2007)
*Recherches sur le probléme de quatre nombres positives et en proportion arithmétique tels, que la somme de deux quelconques soit toujours un nombre carré.* (Research into the problem of four positive numbers and such an arithmetic proportion of them, that the sum of any two of them is always a square number.)
Originally published in *Commentationes arithmeticae* 2, 1849, pp. 617-625 ; *Opera Omnia*: Series 1, Volume 5, pp. 340 - 352
(continues from E796, translated by Kristen McKeen and Thomas Osler in 2006)

[1]http://www.physik.uni-heidelberg.de/fakultaet/historisches/

# Translating Euler and Undergraduate Research

## Why Should Students Translate Euler?

- Euler’s works are accessible, voluminous, diverse, significant, interesting, and written in common languages that are traditionally taught in high school or college: Latin, French, and German

- Mathematical content is accessible to students who’ve had first-year calculus and basic knowledge of Latin, French, or German

- Translating mathematics provides a good transition to research in mathematics by training students to deeply understand the mathematics they are studying before attempting to generate new mathematics through generalization

- *``Read Euler, read Euler. He is the master of us all.'‘* -- Laplace

# Getting Your Students' Feet Wet

## Recruiting

Recruited by word-of-mouth strong junior/senior math majors, including students enrolled in upper-level math courses (History of Math)

## Training

Invited William Dunham (Muhlenberg College) and Edward Sandifer (Western Connecticut State University) to speak at Rowan

## Selecting a Research Topic

Allow students the opportunity to select which work of Euler to translate; otherwise, help student to select an appropriate one if needed.

## Translation Process

Students do the translation essentially on their own or with help from outside sources

## Research Process

Meet with each student once a week to go over the mathematics contained in the translation; process can be very slow, but highly rewarding for both student and faculty advisor

## Writing Up

Write a synopsis of the translation explaining the mathematics for a modern audience

# Disseminating Results

Give talks at regional and national conferences

- Garden State Undergraduate Math Conference
- MathFest
- Joint Math Meetings

# What's Next?

## Expand STEM Project

Recruit additional faculty and students (not necessarily from Rowan) to become involved

## Euler REU

- Submit proposal this summer for NSF Summer REU grant
- Run 8-week undergraduate research program at Rowan University
- Recruit 8 students to translate Euler's works, culminating in translations posted on the Euler Archive and articles suitable for publication in expository math journals
- Program would be appropriate for strong freshmen and sophomore math majors

# References

1. The Euler Archive: http://www.math.dartmouth.edu/~euler/

2. Student Translations of Euler's Mathematics (STEM): http://www.rowan.edu/colleges/las/departments/math/facultystaff/nguyen/euler/index.html

3. Leonhard Euler und Christian Goldbach: briefwechsel 1729-1764. Berlin: Akademie-Verlag, 1965.