

Clinic Consultant Module Spring 19

Quiz 3

Student Name:

1. True or False

a) A fair coin is tossed five times. You will have an equal chance to get HTTHH and HHHHH (T or F)

Each toss is independent with P=1/2. Thus, P(HTTHH)=P(HHHHH)=(1/2)^5

b) A steel manufacturer is testing a new additive for manufacturing an alloy of steel. The joint probability mass function of tensile strength (in thousands of product/n^2) and additive concentration is as follow. Let X be additive concentration of an alloy of steel and Y tensile strength. X and Y are independent (T or F)

Concentration of Additive	Tensile Strength		
	100	150	200
0.02	0.05	0.06	0.11
0.04	0.01	0.08	0.10
0.06	0.04	0.08	0.17
0.08	0.04	0.14	0.12

(a) The marginal probability mass function p\_X(x) is found by summing along the rows of the joint probability mass function.

x	y				p_X(x)
	0	1	2	3	
0	0.15	0.12	0.11	0.10	0.48
1	0.09	0.07	0.05	0.04	0.25
2	0.06	0.05	0.04	0.02	0.17
3	0.04	0.03	0.02	0.01	0.10
p_Y(y)	0.34	0.27	0.22	0.17	

p\_X(0) = 0.48, p\_X(1) = 0.25, p\_X(2) = 0.17, p\_X(3) = 0.10, p\_X(x) = 0 if x ≠ 0, 1, 2, or 3

(b) The marginal probability mass function p\_Y(y) is found by summing down the columns of the joint probability mass function. So p\_Y(0) = 0.34, p\_Y(1) = 0.27, p\_Y(2) = 0.22, p\_Y(3) = 0.17, p\_Y(y) = 0 if y ≠ 0, 1, 2, or 3

c) Scores on a standardized test are approximately normally distributed with a mean of 480 and a standard deviation of 90, the proportion of the scores are above 700 is 0.0073 (T or F)

Normalize the PDF z=(700-480)/90

(a) z = (700 - 480)/90 = 2.44. The area to the right of z = 2.44 is 0.0073.

d) Refer to the same problem 1(c), the 25th percentile of the score is 418 (T or F)

(b) The z-score of the 25th percentile is ≈ -0.67.

The 25th percentile is therefore ≈ 480 - 0.67(90) = 419.7.

2. If X ~ N(2,9), what is P (1 < X < 7) (hint: using Gaussian Table):

- a) 0.5818
- b) 0.9525
- c) 0.0668
- d) 0.4565

First normalize, then looking for table.

For 1,  $z = (1 - 2)/\sqrt{9} = -0.33$ . For 7,  $z = (7 - 2)/\sqrt{9} = 1.67$ .

The area between  $z = -0.33$  and  $z = 1.67$  is  $0.9525 - 0.3707 = 0.5818$ .

3. **Let  $Z \sim N(0, 1)$ . What is the constant  $c$ , so  $P(|Z| \geq c) = 0.1470$**

- a) 2  
b) 1.45  
c) 1  
d) 1.35

$P(|Z| \geq c) = P(Z > c \text{ and } Z < -c) = 0.1470 \Rightarrow P(Z < -c) = 1 - 0.1470/2 = 0.9265$ . According to Table,  $c = 1.45$

4. **The repair time (in hours) for a certain machine is a random variable with probability density function as follows, what is the mean repair time?**

$$f(x) = \begin{cases} xe^{-x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

- a) 1  
b) 2.5  
c) 1.5  
d) 2

Mean = integral of  $\int_0^{\infty} x * xe^{-x} = 2$

5. **Two fair dice are rolled. Let X represent the number on the first dice and Y the number on the second dice. What is  $\mu_{xy}$ ?**

- a) 3.5  
b) 12.25  
c) 7  
d) None of the above

Mean of X =  $1*1/6 + 2*1/6 + 3*1/6 + \dots + 6*1/6 = 3.5$  = mean of Y. The answer =  $3.5 * 3.5$