

Homework #2

Due 11:59pm (Section 2/Section 3) Feb. 8, 2019

You are encouraged to read Chapter 2 of your textbook to complete the homework.

Please follow the name convention to name your **folders and files**: **CC_<lastName>_HW<#>.pdf**. **Your HW will not be graded if you do not follow this name convention.** And electronically submit your HW to **cv094021@gmail.com**

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- (a) False
- (b) True
- (c) True. This is the definition of probability.

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- (a) Since 562 stones were neither cracked nor discolored, 38 stones were cracked, discolored, or both. The probability is therefore $38/600 = 0.0633$.
- (b) Let A be the event that the stone is cracked and let B be the event that the stone is discolored. We need to find $P(A \cap B)$. We know that $P(A) = 15/600 = 0.025$ and $P(B) = 27/600 = 0.045$. From part (a) we know that $P(A \cup B) = 38/600$.
Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Substituting, we find that $38/600 = 15/600 + 27/600 - P(A \cap B)$. It follows that $P(A \cap B) = 4/600 = 0.0067$.
- (c) We need to find $P(A \cap B^c)$. Now $P(A) = P(A \cap B) + P(A \cap B^c)$ (this can be seen from a Venn diagram). We know that $P(A) = 15/600$ and $P(A \cap B) = 4/600$. Therefore $P(A \cap B^c) = 11/600 = 0.0183$.

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- (a) $(26^3)(10^3) = 17,576,000$
- (b) $(26)(25)(24)(10)(9)(8) = 11,232,000$
- (c) $\frac{11,232,000}{17,576,000} = 0.6391$

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Let A denote the event that the allocation sector is damaged, and let N denote the event that a non-allocation sector is damaged. Then $P(A \cap N^c) = 0.20$, $P(A^c \cap N) = 0.70$, and $P(A \cap N) = 0.10$.

$$(a) P(A) = P(A \cap N^c) + P(A \cap N) = 0.3$$

$$(b) P(N) = P(A^c \cap N) + P(A \cap N) = 0.8$$

$$\begin{aligned}(c) P(N|A) &= \frac{P(A \cap N)}{P(A)} \\ &= \frac{P(A \cap N)}{P(A \cap N) + P(A \cap N^c)} \\ &= \frac{0.10}{0.10 + 0.20} \\ &= 1/3\end{aligned}$$

$$\begin{aligned}(d) P(A|N) &= \frac{P(A \cap N)}{P(N)} \\ &= \frac{P(A \cap N)}{P(A \cap N) + P(A^c \cap N)} \\ &= \frac{0.10}{0.10 + 0.70} \\ &= 1/8\end{aligned}$$

$$\begin{aligned}(e) P(N^c|A) &= \frac{P(A \cap N^c)}{P(A)} \\ &= \frac{P(A \cap N^c)}{P(A \cap N^c) + P(A \cap N)} \\ &= \frac{0.20}{0.20 + 0.10} \\ &= 2/3\end{aligned}$$

Equivalently, one can compute $P(N^c|A) = 1 - P(N|A) = 1 - 1/3 = 2/3$

$$\begin{aligned}(f) P(A^c|N) &= \frac{P(A^c \cap N)}{P(N)} \\ &= \frac{P(A^c \cap N)}{P(A^c \cap N) + P(A \cap N)} \\ &= \frac{0.70}{0.70 + 0.10} \\ &= 7/8\end{aligned}$$

Equivalently, one can compute $P(A^c|N) = 1 - P(A|N) = 1 - 1/8 = 7/8$

$$(a) \frac{71}{102 + 71 + 33 + 134} = \frac{71}{340}$$

$$(b) \frac{86}{102 + 86 + 26} = \frac{43}{107}$$

$$(c) \frac{22}{26 + 32 + 22 + 40} = \frac{11}{60}$$

$$(d) \frac{22}{33 + 36 + 22} = \frac{22}{91}$$

$$(e) \frac{86 + 63 + 36 + 26 + 32 + 22}{86 + 63 + 36 + 105 + 26 + 32 + 22 + 40} = \frac{53}{82}$$

A chain of stores sells three different cell phones. Of its sales, 50% are brand 1, 30% are brand 2, 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1's cell phones require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

1. What is the probability that a randomly selected purchaser has brought Brand 1's cell phones that will need repair under warranty?
2. What is the probability that a randomly selected purchaser that will need repair under warranty
3. If a customer returns to the store with a cell phone that needs warranty repair work, what is the probability that it is a brand 1 cell phones? A brand 2 cell phones? A brand 3 cell phones?

[Solution]:

$$P(A1)=P(\text{purchased Brand1 cell})=0.5$$

$$P(A2)=P(\text{purchased Brand2 cell})=0.3$$

$$P(A3)=P(\text{purchased Brand3 cell})=0.2$$

Event B=phone needs repair

$$P(B|A1)=0.25; P(B|A2)=0.2; P(B|A3)=0.1$$

$$(1) P(A1 \cap B)=P(B|A1)P(A1)=0.25*0.5=0.125$$

(2) $P(B)$

= $P[(\text{brand 1 and need repair}) \text{ or } (\text{brand 2 and need repair}) \text{ or } (\text{brand 3 and need repair})]$ = $P(A1 \cap B) + P(A2 \cap B) + P(A3 \cap B) = 0.205$

(3) $P(A1|B) = P(A1 \cap B) / P(B) = 0.125 / 0.205 = 0.609$

$P(A2|B) = P(A2 \cap B) / P(B) = 0.06 / 0.205 = 0.293$

$P(A3|B) = P(A3 \cap B) / P(B) = 0.02 / 0.205 = 0.098$