Homework #3

Due 12:15pm/13:45pm (Section 2/Section 3) Feb. 25, 2019

You are encouraged to read Chapter 4 of your textbook to complete the homework

Pages 202-203:

2. (a) $p_X = 0.2$

   (b) $p_Y = 0.45$

   (c) $p_Z = 0.65$

   (d) No. If the order is for a red set, it cannot also be for a white set.

   (e) Yes. $p_Z = 0.65 = 0.2 + 0.45 = p_X + p_Y$.

   (f) Yes. If the set is red, then $X = 1$, $Y = 0$, and $Z = 1$, so $Z = X + Y$. If the set is white, then $X = 0$, $Y = 1$, and $Z = 1$, so $Z = X + Y$. If the set is blue, then $X = 0$, $Y = 0$, and $Z = 0$, so $Z = X + Y$.

4. (a) If X and Y cannot both be equal to 1, the possible values for Z are 0 and 1, so Z is a Bernoulli random variable.

   (b) $p_Z = P(Z = 1) = P(X = 1 \text{ or } Y = 1) = P(X = 1) + P(Y = 1) - P(X = 1 \text{ and } Y = 1) = P(X = 1) + P(Y = 1) - 0 = p_X + p_Y$

   (c) $P(Z = 2) = P(X = 1 \text{ and } Y = 1) \neq 0$. Since $P(Z = 2) \neq 0$, Z is not a Bernoulli random variable.

Pages 212-215: 2, 8 (a), (c) and (e), 16 (a), (d)

2. (a)
$$\begin{aligned} P(X > 6) &= P(X = 7) + P(X = 8) + P(X = 9) \\ &= \frac{9!}{7!(9-7)!}(0.4)^7(1-0.4)^{9-7} + \frac{9!}{8!(9-8)!}(0.4)^8(1-0.4)^{9-8} + \frac{9!}{9!(9-9)!}(0.4)^9(1-0.4)^{9-9} \\ &= 0.0250 \end{aligned}$$

(b)
$$\begin{aligned} P(X \geq 2) &= 1 - P(X < 2) \\ &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - \frac{9!}{0!(9-0)!}(0.4)^0(1-0.4)^{9-0} - \frac{9!}{1!(9-1)!}(0.4)^1(1-0.4)^{9-1} \\ &= 0.9295 \end{aligned}$$

$$\begin{aligned}
\text{(c) } P(2 \le X < 5) &= P(X=2)+P(X=3)+P(X=4) \\
&= \frac{9!}{2!(9-2)!}(0.4)^2(1-0.4)^{9-2} + \frac{9!}{3!(9-3)!}(0.4)^3(1-0.4)^{9-3} + \frac{9!}{4!(9-4)!}(0.4)^4(1-0.4)^{9-4} \\
&= 0.6629
\end{aligned}$$

$$\begin{aligned}
\text{(d) } P(2 < X \le 5) &= P(X=3)+P(X=4)+P(X=5) \\
&= \frac{9!}{3!(9-3)!}(0.4)^3(1-0.4)^{9-3} + \frac{9!}{4!(9-4)!}(0.4)^4(1-0.4)^{9-4} + \frac{9!}{5!(9-5)!}(0.4)^5(1-0.4)^{9-5} \\
&= 0.6687
\end{aligned}$$

(e) $P(X=0) = \frac{9!}{0!(9-0)!}(0.4)^0(1-0.4)^{9-0} = 0.0101$

(f) $P(X=7) = \frac{9!}{7!(9-7)!}(0.4)^7(1-0.4)^{9-7} = 0.0212$

(g) $\mu_X = (9)(0.4) = 3.6$

(h) $\sigma_X^2 = (9)(0.4)(0.6) = 2.16$

8. Let X be the number of contracts, out of the 20 chosen, that have a cost overrun. Then $X \sim \text{Bin}(20, 0.2)$.

(a) $P(X=4) = \frac{20!}{4!(20-4)!}(0.2)^4(1-0.2)^{20-4} = 0.2182$

(c) $P(X=0) = \frac{20!}{0!(20-0)!}(0.2)^0(1-0.2)^{20-0} = 0.0115$

(e) $\sigma = \sqrt{(20)(0.2)(1-0.2)} = 1.7889$

16. (a) Let X be the number of components out of 10 that are defective. Then $X \sim \text{Bin}(10, 0.1)$.

$$\begin{aligned}
P(\text{return}) &= P(X>1) \\
&= 1-P(X\le 1) \\
&= 1-P(X=0)-P(X=1) \\
&= 1-\frac{10!}{0!(10-0)!}(0.1)^0(1-0.1)^{10-0} - \frac{10!}{1!(10-1)!}(0.1)^1(1-0.1)^{10-1} \\
&= 1-0.34868-0.38742 \\
&= 0.2639
\end{aligned}$$

(d) Let n be the required sample size. Let X be the number of sampled components that are defective. Then $X \sim \text{Bin}(n, 0.2)$.

$P(\text{accept}) = P(X=0) = (0.8)^n \le 0.01$. So $n \ln 0.8 \le \ln 0.01$, $-0.2231n \le -4.605$, $n \ge 20.64$. The minimum sample size is 21.