Homework #4

Due 11:59pm (Section 2/Section 3) March 1, 2019

You are encouraged to read Chapter 4 of your textbook to complete the homework.

Please follow the name convention to name your **folders and files**: *CC_<lastName>_HW<#>.pdf*. **Your HW will not be graded if you do not follow this name convention. And** electronically submit your HW to cv094021@gmail.com

Pages 227-229: 6, 12, 16

6. Let X be the number of individuals in the sample that carry the gene. Then X is the number of successes in $n = 1000$ Bernoulli trials, each of which has success probability $p = 0.0002$. The mean of X is $np = (1000)(0.0002) = 0.2$. Since n is large and p is small, $X \sim \text{Poisson}(0.2)$ to a very close approximation.

(a) $P(X = 1) = e^{-0.2}\dfrac{0.2^1}{1!} = 0.1637$

(b) $P(X = 0) = e^{-0.2}\dfrac{0.2^0}{0!} = 0.8187$

(c)
$$\begin{aligned} P(X > 2) &= 1 - P(X \le 2) \\ &= 1 - P(X = 0) - P(X = 1) - P(X = 2) \\ &= 1 - e^{-0.2}\frac{0.2^0}{0!} - e^{-0.2}\frac{0.2^1}{1!} - e^{-0.2}\frac{0.2^2}{2!} \\ &= 1 - 0.81873 - 0.16375 - 0.016375 \\ &= 0.0011 \end{aligned}$$

(d) Since $X \sim \text{Poisson}(0.2)$, $\mu_X = 0.2$.

(e) Since $X \sim \text{Poisson}(0.2)$, $\sigma_X = \sqrt{0.2} = 0.45$.

12. (a) Let X be the number of plants in a two-acre region. Then $X \sim \text{Poisson}(20)$.

$$P(X = 18) = e^{-20}\frac{20^{18}}{18!} = 0.08439$$

(b) The area of the circle is $10,000\pi = 31,415.9$ ft^2. In units of acres, the area is $31,415.9/43,560 = 0.72121$ acres. Let X be the number of plants in the circle. Then $X \sim \text{Poisson}(7.2121)$.

$$P(X = 12) = e^{-7.2121}\frac{7.2121^{12}}{12!} = 0.0305$$

(c) Let Y represent the number of plants observed in 0.1 acres. Let λ represent the true concentration in plants per acre. Then $Y \sim \text{Poisson}(0.1\lambda)$. The observed value of Y is 5. The estimated concentration is $\widehat{\lambda} = 5/0.1 = 50$. The uncertainty is $\sigma_{\widehat{\lambda}} = \sqrt{50/0.1} = 22.36$. $\lambda = 50 \pm 22$.

16. (a) Let X be the number of raisins in a randomly chosen slice.

Since the mean number of raisins per slice is $100/60 = 5/3$, $X \sim \text{Poisson}(5/3)$.

$P(X = 0) = e^{-(5/3)}\dfrac{(5/3)^0}{0!} = 0.1889$

(b) Let X be the number of raisins in a randomly chosen slice.

Since the mean number of raisins per slice is $200/60 = 10/3$, $X \sim \text{Poisson}(10/3)$.

$P(X = 5) = e^{-(10/3)}\dfrac{(10/3)^5}{5!} = 0.1223$

(c) Let n be the required number of raisins. Let X be the number of raisins in a randomly chosen slice.

Then $X \sim \text{Poisson}(n/60)$.

$P(X = 0) = e^{-(n/60)}\dfrac{(n/60)^0}{0!} = e^{-(n/60)} = 0.01$

$-n/60 = \ln 0.01 = -4.605$, so $n = 276.3$

The smallest value of n for which $P(X = 0) \leq 0.01$ is $n = 277$.