Homework #5

Due 11:59pm (Section 2/Section 3) March 20, 2019

You are encouraged to read Chapters 2 and 4 of your textbook to complete the homework

Please follow the name convention to name your **folders and files**: *CC_<lastName>_HW<#>.pdf*. **Your HW will not be graded if you do not follow this name convention. And** electronically submit your HW to cv094021@gmail.com

Pages 114-116:

18. (a) $P(X > 2) = \int_2^{\infty} \frac{64}{(x+2)^5}\,dx = -\frac{16}{(x+2)^4}\Big|_2^{\infty} = 1/16$

(b) $P(2 < X < 4) = \int_2^{4} \frac{64}{(x+2)^5}\,dx = -\frac{16}{(x+2)^4}\Big|_2^{4} = 65/1296$

(c) $\mu = \int_0^{\infty} x\frac{64}{(x+2)^5}\,dx = \int_2^{\infty} (u-2)\frac{64}{u^5}\,du = 64\int_2^{\infty} (u^{-4} - 2u^{-5})\,du = 64\left(-\frac{1}{3}u^{-3} + \frac{1}{2}u^{-4}\right)\Big|_2^{\infty} = 2/3$

(d) $$\begin{aligned}\sigma^2 &= \int_0^\infty x^2 \frac{64}{(x+2)^5}\,dx - \mu^2 \\ &= \int_2^\infty (u-2)^2 \frac{64}{u^5}\,du - (2/3)^2 \\ &= 64\int_2^\infty (u^{-3} - 4u^{-4} + 4u^{-5})\,du \quad - \quad 4/9 \\ &= 64\left(-\frac{1}{2}u^{-2} + \frac{4}{3}u^{-3} - u^{-4}\right)\Bigg|_2^\infty \quad - \quad 4/9 \\ &= 8/9\end{aligned}$$

(e) $F(x) = \int_{-\infty}^x f(t)\,dt$.

If $x < 0$, $F(x) = \int_{-\infty}^x 0\,dt = 0$.

If $x \geq 0$, $F(x) = \int_0^x \frac{64}{(t+2)^5}\,dt = -\frac{16}{(t+2)^4}\Bigg|_0^x = 1 - \frac{16}{(x+2)^4}$

(f) The median x_m solves $F(x_m) = 0.5$. Therefore $1 - \frac{16}{(x_m+2)^4} = 0.5$, so $x_m = 0.3784$.

(g) The 60th percentile x_{60} solves $F(x_{60}) = 0.6$. Therefore $1 - \frac{16}{(x_{60}+2)^4} = 0.6$, so $x_{60} = 0.5149$.

24. (a) c solves the equation $\int_1^\infty c/x^3\,dx = 1$. Therefore $-0.5c/x^2\Bigg|_1^\infty = 1$, so $c = 2$.

(b) $\mu_X = \int_1^\infty cx/x^3\,dx = \int_1^\infty 2/x^2\,dx = -\frac{2}{x}\Bigg|_1^\infty = 2$

(c) $F(x) = \int_{-\infty}^x f(t)\,dt$

If $x < 1$, $F(x) = \int_{-\infty}^{x} 0\,dt = 0$.

If $x \geq 1$, $F(x) = \int_{-\infty}^{1} 0\,dt + \int_{1}^{x} 2/t^3\,dt = -\frac{1}{t^2}\Big|_1^x = 1 - 1/x^2$.

(d) The median x_m solves $F(x_m) = 0.5$. Therefore $1 - 1/x_m^2 = 0.5$, so $x_m = 1.414$.

(e) $P(X \leq 10) = F(10) = 1 - 1/10^2 = 0.99$

(f) $P(X \leq 2.5) = F(2.5) = 1 - 1/2.5^2 = 0.84$

(g) $P(X \leq 2.5 | X \leq 10) = \frac{P(X \leq 2.5 \text{ and } X \leq 10)}{P(X \leq 10)} = \frac{P(X \leq 2.5)}{P(X \leq 10)} = \frac{0.84}{0.99} = 0.85$

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2. (a) Using Table A.2: 0.7123

(c) Using Table A.2: $0.7580 - 0.1401 = 0.6179$

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(b) For 1, $z = (1-2)/\sqrt{9} = -0.33$. For 7, $z = (7-2)/\sqrt{9} = 1.67$.
The area between $z = -0.33$ and $z = 1.67$ is $0.9525 - 0.3707 = 0.5818$.

(d) $P(-3 \leq X - 2 < 3) = P(-1 \leq X < 5)$.
For -1, $z = (-1-2)/\sqrt{9} = -1.00$. For 5, $z = (5-2)/\sqrt{9} = 1.00$.
The area between $z = -1.00$ and $z = 1.00$ is $0.8413 - 0.1587 = 0.6826$.

8. (a) For 3.7, $z = (3.7 - 4.1)/0.6 = -0.67$. For 4.4, $z = (4.4 - 4.1)/0.6 = 0.50$.
The area between $z = -0.67$ and $z = 0.50$ is $0.6915 - 0.2514 = 0.4401$.

(d) $z = (4.5 - 4.1)/0.6 = 0.67$. The area to the right of $z = 0.67$ is 0.2514.

(e) Let X be the number of cats that weigh more than 4.5 kg. Using part (d), the probability that a cat weighs more than 4.5 kg is 0.2514. Therefore $X \sim \text{Bin}(6, 0.2514)$.

$P(X = 1) = \frac{6!}{1!(6-1)!}(0.2514)^1(1 - 0.2514)^{6-1} = 0.3546$.

20. (a) The z-score of 160 is $(160 - 200)/10 = -4.00$. The area to the left of $z = -4.00$ is negligible. Therefore $P(X \leq 160) \approx 0$.

(b) Yes. If the procedure is functioning correctly, values of 160 N or less would almost never occur.

(c) Yes, because 160 N is unusually small if the process is functioning correctly.