

Homework #3

1. (2 point) Please formally define the vending machine's FSM in Fig. 1, i.e., Σ , Q , q_0 , δ , and F

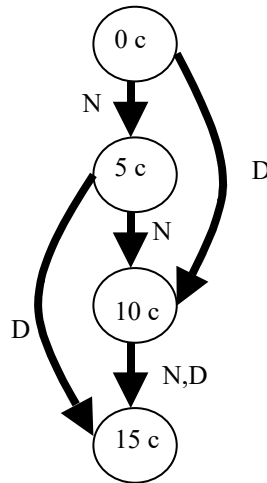


Fig. 1

Solution:

$$\Sigma = \{ \text{receiving } 0, \text{ receiving } N \} = \{ 0, 1 \}$$

$$Q = \{ q_0, q_1, q_2, q_3 \} = \{ 0, 5, 10, 15 \}$$

$$q_0 = 0$$

$$\delta : \begin{array}{ll} \delta(q_0, 0) = q_2 & \delta(q_0, 1) = q_1 \\ \delta(q_1, 0) = q_3 & \delta(q_1, 1) = q_2 \\ \delta(q_2, 0) = q_3 & \delta(q_2, 1) = q_3 \end{array}$$

$$F = \{ \emptyset \}$$

2. (2 point) Please derive L and L_m of the FSM in Fig. 1 if the state 15c is a final state.

Solution:

$$L = \{\epsilon, 0, 01, 00, 000, 001, 1, 10, 11\}$$

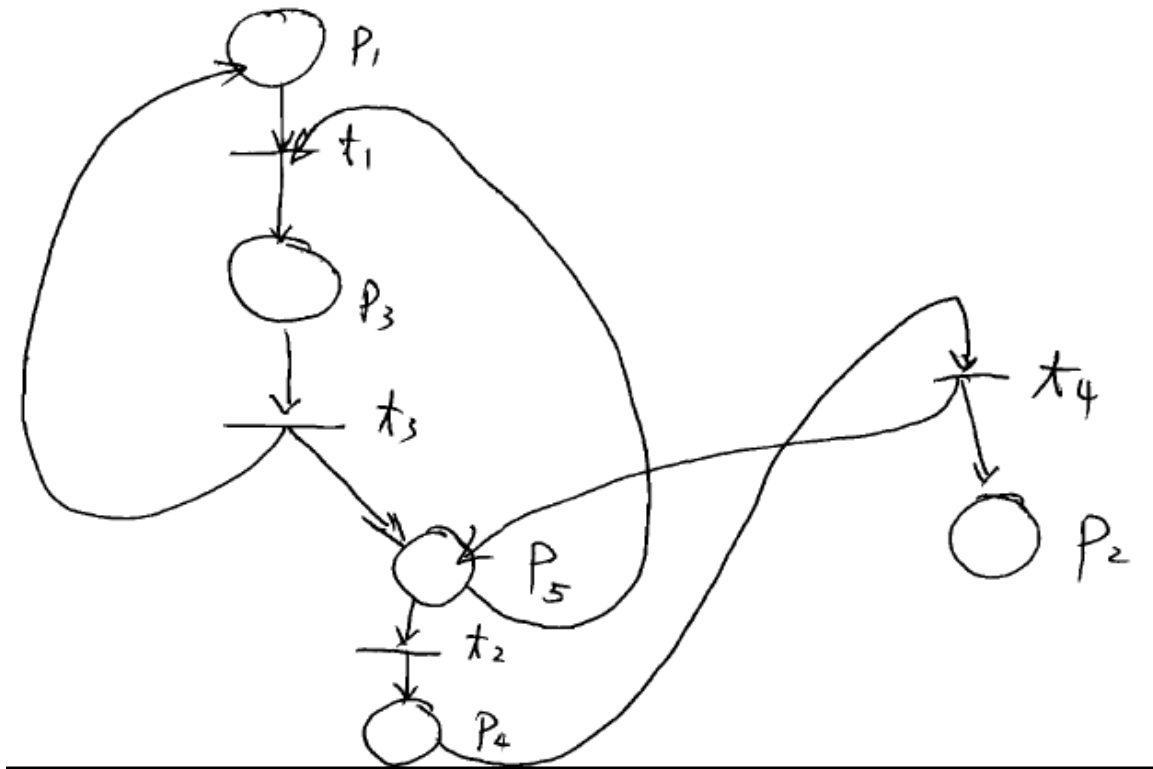
$$L_m = \{01, 000, 10, 11, 001\}$$

3. (2 point) Represent graphically the following Petri net (P, T, I, O) where $P = \{p_i, 1 \leq i \leq 5\}$ and $T = \{t_i, 1 \leq i \leq 4\}$,

$$I = \begin{pmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \\ 1100 \end{pmatrix} \quad O = \begin{pmatrix} 0010 \\ 0001 \\ 1000 \\ 0100 \\ 0011 \end{pmatrix}$$

Solution

P2 is the input of t2 (which is missing in the solution)



4. (5 points) Please derive the reachability tree of the PN in Fig. 2, and determine its boundedness, safeness, liveness, and reversibility. J is the last digit of your banner ID. For $i=1, 2, \dots, 9$, $a_i=0$ if your i -th ID digit is smaller than or equal to 5, and otherwise 1. For example, if you ID =121291755, $a_1-a_9=000010100$. Let

$m_0(p_1)=a_6$, $m_0(p_2)=(1-a_6)$; $m_0(p_6)=a_7$, $m_0(p_3)=(J+2)*(1-a_7)$; $m_0(p_4)=a_8$, $m_0(p_5)=1-a_8$.

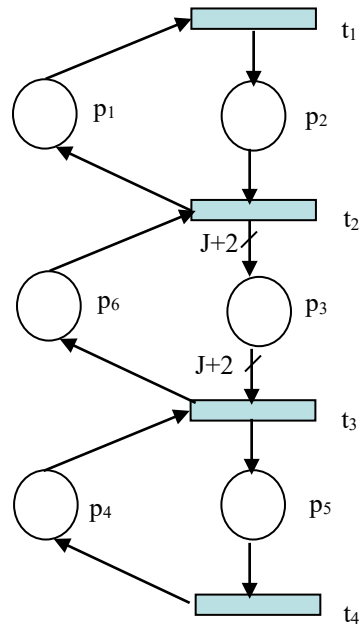


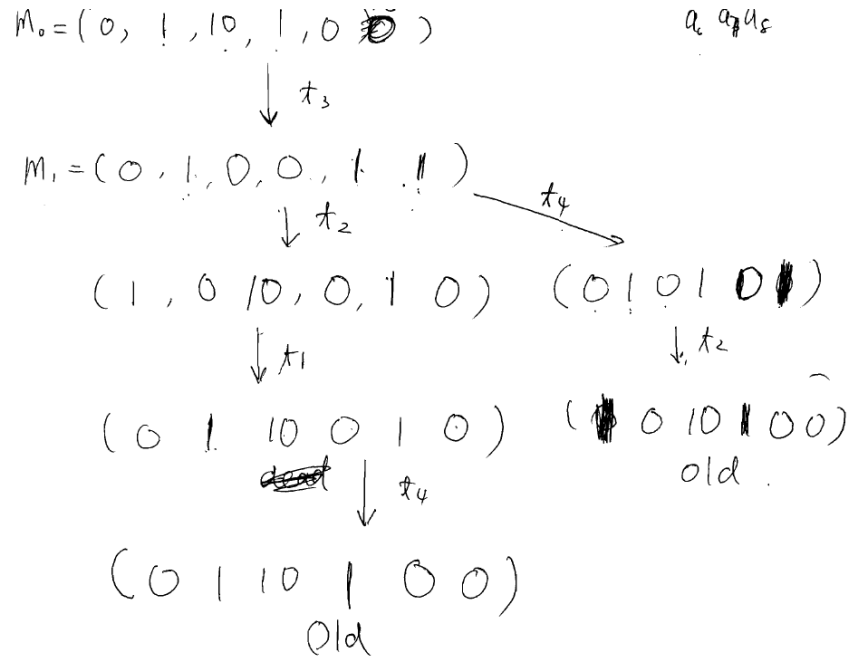
Fig. 2

Solution:

For this solution the ID used is 91008328

Thus, $m_0 = \{0, 1, 10, 1, 0, 0\}$

Thus, the reachability graph is as follows:



It is bounded, but not safe, and live, and reversible

5. (5 points) Please derive the reachability tree of the following PN in Fig. 3 and its boundedness, safeness, liveness, and reversibility. HINT: you may not be able to judge its properties based on the tree. Need more thinking and extra work.

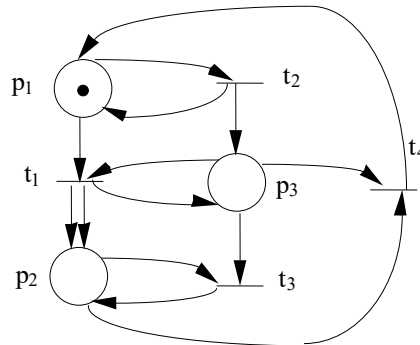


Fig. 3

Solution

$$m_0 = (1 \ 0 \ 0)$$

~~t_1~~

t_2

$$m_1 = (\text{█}, 0 \ 1)$$

$$\forall p \ m_1(p) \geq m_0(p) = (1, 0, \omega)$$

$$m_1(p_3) > m_0(p_3)$$

\therefore Chang $m_1(p_3) = \omega$

t_1

t_2

$$m_2 = (0, 2, \omega)$$

t_4

t_3

$$m_3 = (1, 1, \omega) \quad (0, 2, \omega)$$

~~t_4~~ old

$\forall p \ m_3(p) \geq m_i(p) \ (i=0, 1)$
 $m_3(p_2) > m_0(p_2)/m_1(p_2)$
 Chang $m_3(p_2) = \omega$

$$m_3 = (1, \omega, \omega)$$

t_1

t_2

t_3

t_4

$$m_5 = (0, \omega, \omega)$$

$$(1, \omega, \omega)$$

old

old

$$m_4 = (2, \omega, \omega)$$

~~t_4~~

t_3

$$(0, \omega, \omega)$$

old

$\forall p \ m_4(p) \geq m_3(p)$
 $m_4(p_1) > m_3(p_1)$

$m_6 = (\omega, \omega, \omega)$ Chang $m_4(p) = \omega$

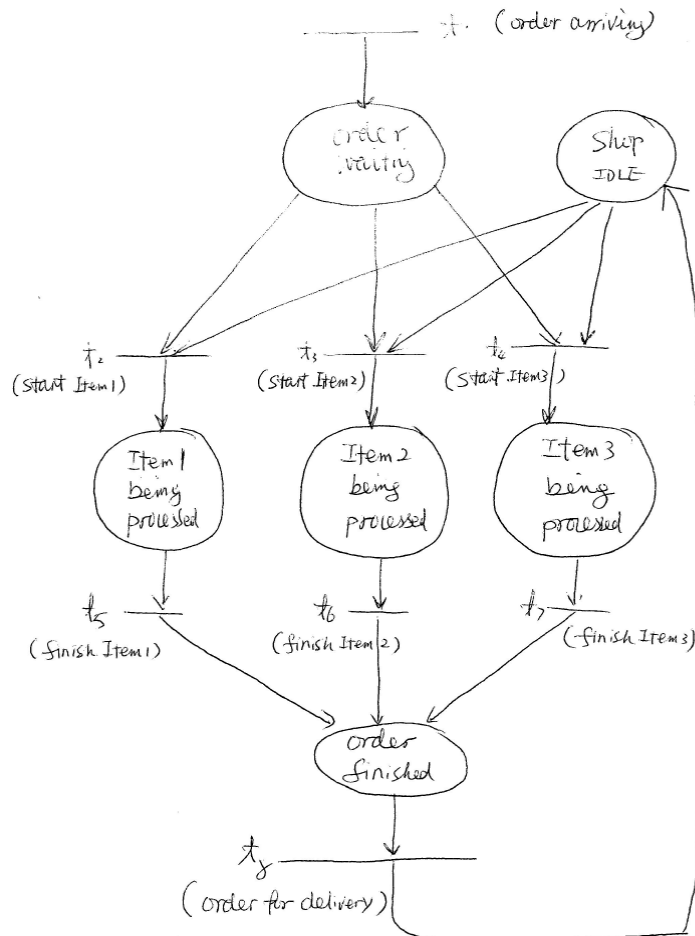
$$(1, \omega, \omega)$$

old

\therefore it is not bounded, not safe, not reversible
 but live (some transitions can only fire
 once in a firing seq. thus they are LI-live)
 \therefore The PN is LI-live

6. (4 points). A small machine shop can make three different items, though it can work on only one item at a time. The shop can be in six different “states” corresponding to: an order is waiting, one of the three items is being made, an order is finished, and the shop is idle. There are six actions: order arriving, start work on item 1, start work on item 2, start work on item 3, finish processing, and order sent for delivery. Construct a Petri net model for this machine shop?

Solution:



7. (4 points) Construct a PN model for calculating the logic conjunction of two variables x and y , each of which takes the values of “true” and “false”. Each is assigned a value independently of the other.

