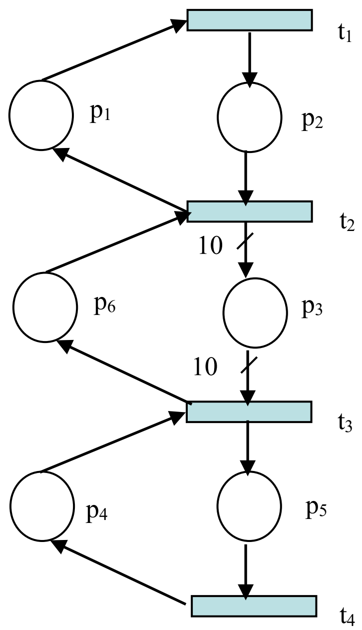
Homework #4

Due 11:59pm April 19th, 2019

1. Find the incidence matrix of the PN in the following figure, then its independent P and T-invariants; and positive P and T-invariants



Solution:

$$A = O - I = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & +10 & -10 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix}$$

$$A\mu = 0 \Rightarrow \begin{aligned} -\mu_1 + \mu_2 &= 0 \\ \mu_2 - \mu_3 &= 0 \\ -\mu_3 + \mu_4 &= 0 \end{aligned} \Rightarrow \begin{aligned} \mu_1 &= \mu_2 \\ \mu_2 &= \mu_3 \\ \mu_3 &= \mu_4 \end{aligned}$$

$\mu_1 = \mu_2 = \mu_3 = \mu_4 = 1$ T-invariant $\{1, 1, 1, 1\}$

$$A^T x = 0$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 10 & 0 & 0 & -1 \\ 0 & 0 & -10 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = 0$$

- ➔ x1=x2
- ➔ 10x3=x6
- ➔ x4=x5

Let x1=1; x3=1; x4=1, then x2=1, x6=10, x5=1

We got the P-invariant as (1, 1, 1, 1, 1, 10)

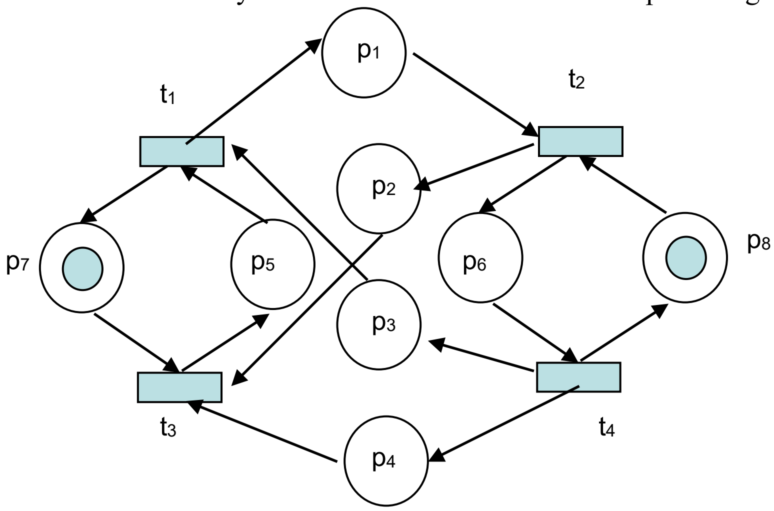
We can random change x1, x3, and x4, then get different P-invariant:

X1	X3	X4	X2	X6	X5
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	10	0
0	1	1	0	10	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	10	0
1	1	1	1	10	1

Note that if you chose x >1, you will find the resulting p-invariant is dependent on some of the p-invariant when choosing x=1.

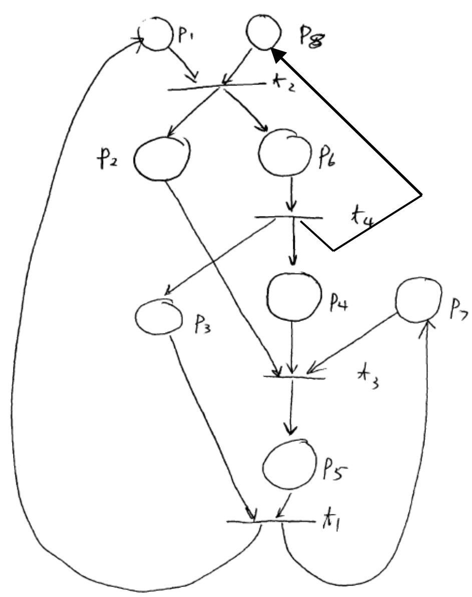
2. For the timed PN in the following figure, let the delay of transition i and place i be x_i, i=1 to 8. **x_i=the i-th digit of your banner ID.** Delay i+j is associated with arc (pi, tj) but none with arc (tj, pi), for i=1, 2,…, 8 and j=1, 2, …, 4. Places p2, p4-p8 are each initially marked with one token respectively. p1 and p3 are marked by $1+x_1$ and $1+x_3$ tokens respectively. A) find all the loops and their cycle times and the system cycle time. B) If you are given two more tokens, which place(s) do you want to put

your tokens to reduce the system cycle time? What is the percentage of the improvement? C) If you are allowed to reduce the delay at one place to half, which place will you choose to reduce? What is the percentage of the improvement? D) If you are allowed to reduce the delay at a transition to half, which transition will you choose to reduce? What is the percentage of the improvement?



Solution:

Step (1) Re-draw the PN to figure out the cycles:



Step (2) It is clear from the PN above, there are five cycles:

L_1: $p_1\ t_2\ p_2\ t_3\ p_5\ t_1\ p_1$

L_2: $\cancel{p}_8\ t_2\ p_6$ t4t8

L_3: $p_1\ t_2\ p_6 t_4 p_4\ t_3\ p_5\ t_1\ p_1$

L_4: $p_1\ t_2\ p_6\ t_4\ p_3\ t_1\ p_1$

L_5. $p_7\ t_3\ p_5\ t_1\ p_7$

Step (3) Using the ID = 910008328, we do the following calculations on the cycle time of each loop:

L_1: $\text{delays} = x_1 + x_2 + x_2 + x_3 + x_5 + x_1 + (1+2) + (2+3) + (5+1)$

$= 9+2+2+0+0+9+3+5+6$

$= 36$

$\#$ of tokens $= 10 + 3 = 13$. $\Rightarrow \frac{36}{13} = 2.76$

$L_2 = X_8 + (8+2) + X_2 + X_6 +$ X4 +(6+4)

$= 2 + 10 + 2 + 8 + 0 +$ 10 = 32 $=$ delays

$\#$ of tokens $= 1 + 1 = 2$ $\Rightarrow$ 32/2 =16

$L_3 = X_1 + X_2 + (1+2) + X_6 + X_4 + (4+6) + X_4$

$+ X_3 + (4+3) + X_5 + X_1 + (5+1)$

$= 9+2+3+8+0+10+0+0+7+0+9+6$

$= 54$ $\Rightarrow \frac{54}{13} = 4.14$

$\#$ of tokens $= 10 + 1 + 1 + 1 = 13$

$$L_4: \quad x_1 + x_2 + x_6 + x_4 + x_3 + x_1 + (1+2) + (6+4) + (3+1)$$
$$= 9+2+8+0+0+9+3+10+4$$
$$= 45 \quad \Rightarrow \quad \frac{45}{12} = 3.75$$
$$\#\text{of tokens} = 10+1+1 = 12$$

$$L_5 = \quad x_7 + x_3 + x_5 + x_1 + (7+3) + (5+1)$$
$$= 3+0+0+9+10+6$$
$$= 28 \quad \Rightarrow \quad \frac{28}{2} = 14$$
$$\#\text{of tokens} = 1+1 = 2$$

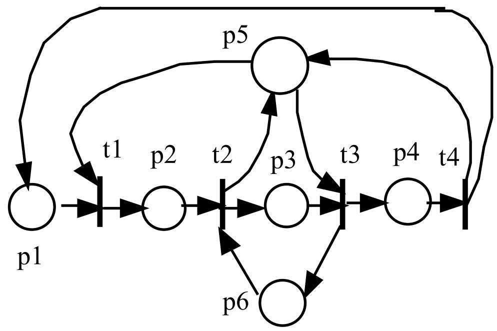
Step (4) The system cycle time = max (cycle time of all loops) = 16

Step (5) For the additional two tokens, we should put them in P8 and P7, respectively, to reduce the system cycle time. When we add one token to p8, it reduces the cycle time of L2 to 10.6. Similarly, when we add one token to p7, it reduces the cycle time of L5 to 9.3
Thus, the system cycle time is reduced to 10.6
The improvement = (16-10.6)/16 *100%

Step (6) Similarly, the goal of reducing one place delay by half is to reduce the system cycle time. The system cycle time is held up by L2 and L5. Based on the current situation, we observe that the best way is to reduce P6 delay, which result in a reduction in L2 cycle time to 14.
Thus, the system cycle time is 14. The improvement = (16-14)/16*100%

Step (7) The best way is to reduce t2 delay by half. The improvement = (16-14)/16*100%

3. Reduce the PN in the following figure given the initial marking m_0=(2 1 0 0 0 4). *Hint: Do not over-do it. Apply only those applicable*



Solution:

The only reduction is to combine t3 and t4 (the two series transitions)

4. The stochastic PN in the following figure is used to model the manufacturing cell where incoming material (p_1) is first transported by R1 (p_7) to M1 (p_5) for processing (p_2 represents M1 is in processing), then loaded to a buffer (p_6), then transported by R 2 (p_9) to M2 (p_8) for processing. M1 may break down. On the average, M1 takes 2 time units to break down and ¼ time unit to be repaired (i.e., the average failure and repair rates are 0.5 and 4, respectively). M2 is failure-free. The loading rate for R1 is 40. The average rate for M1's processing a part plus R1's unloading is 5 per unit time. The average rate for M2's processing plus R2's loading and unloading is 4 per unit time. Calculate the average utilization of M1 and the system throughput if two parts are in the input area?

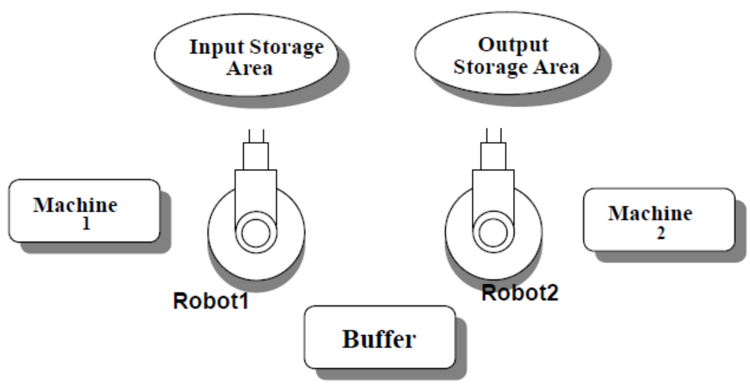


Fig. 2. The manufacturing cell

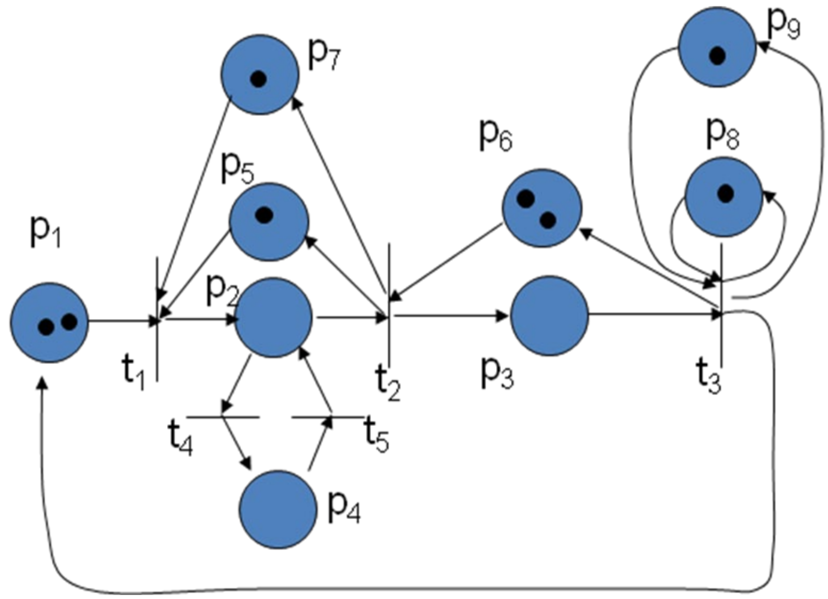


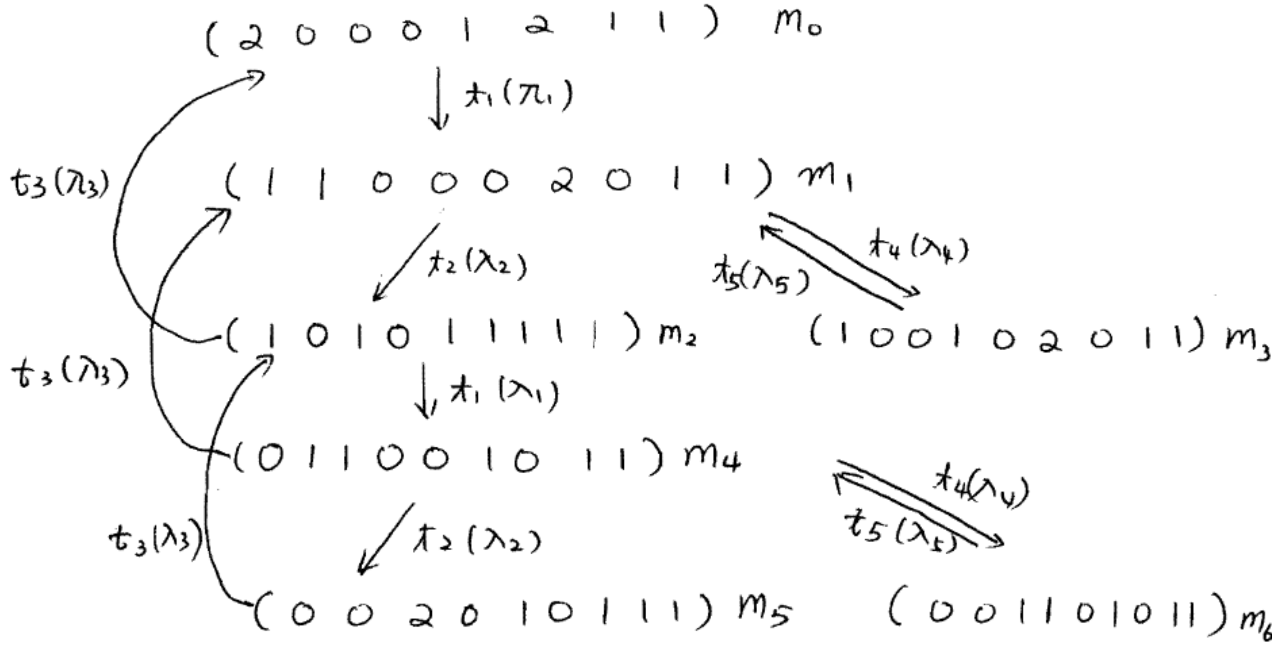
Fig. 3. The PN model for the manufacturing cell in Fig. 2

p1: parts are available
p2: M1 is in processing
p3: Part is ready for M2
p4: M1 is in repair
p5: M1 is available
p6: buffer is available
p7: R1 is available
p8: M2 is available
t5: repair (4)

p9: R2 is available
t1: R1 is loading part to M1 (40)
t2: M1 is processing and R2 is unloading (5)
t3: M2 is processing, R2 is loading & unloading (4)
t4: M1 is breaking down (0.5)

Solution:

Step (1) Reachability graph:



Step (2) Transition equations

$$-\pi_0 \lambda_1 + \lambda_3 \cdot \pi_2 = 0 \quad \text{..... balance equation @ } m_0$$

$$\pi_0 \lambda_1 - \pi_1 \lambda_2 - \pi_1 \lambda_4 + \lambda_3 \pi_4 + \lambda_5 \pi_3 = 0 \quad \text{... balance equ. @ } m_1$$

$$-\pi_2 \lambda_3 - \pi_2 \lambda_1 + \pi_1 \lambda_2 + \pi_5 \lambda_3 = 0 \quad \text{... balance equ. @ } m_2$$

$$-\pi_3 \lambda_5 + \pi_1 \lambda_4 = 0$$

$$-\pi_4 \cdot \lambda_4 - \pi_4 \lambda_2 - \pi_4 \lambda_3 + \pi_2 \lambda_1 + \pi_6 \lambda_5 = 0$$

$$-\pi_5 \lambda_3 + \pi_4 \lambda_2 = 0$$

$$-\pi_6 \lambda_5 + \pi_4 \lambda_4 = 0$$

$$\pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 = 1$$

Step (3) Transition matrix

$$A = \begin{array}{c} \begin{array}{cccccccc} & 40 & 40 & & & & & \end{array} \\ \begin{bmatrix} 1 & -\lambda_1 & \lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -(\lambda_2+\lambda_4) & \lambda_2 & \lambda_4 & 0 & 0 & 0 \\ 1 & \lambda_3 & 0 & -(\lambda_1+\lambda_3) & 0 & \lambda_1 & 0 & 0 \\ 1 & 0 & \lambda_5 & 0 & -\lambda_5 & 0 & 0 & 0 \\ 1 & 0 & \lambda_3 & 0 & 0 & -(\lambda_2+\lambda_3+\lambda_4) & \lambda_2 & \lambda_4 \\ 1 & 0 & 0 & \lambda_3 & 0 & 0 & -\lambda_3 & 0 \\ 1 & 0 & 0 & 0 & 0 & \lambda_5 & 0 & -\lambda_5 \end{bmatrix} \end{array}$$

Step (4) $[\pi 0, \pi 1, \pi 2, \pi 3, \pi 4, \pi 5, \pi 6]A=[0, 0, 0, 0, 0, 0, 0,1]'$

Step (5) Solve the equations for π_i

Step (6) the average utilization of M1 =P2 has a token = ($\pi 1+ \pi 4$)

Step (7) the system throughput = t3 is fired = $(\pi 2+\pi 4+\pi 5)*\lambda 3$