ECE 09468/09568
Discrete Event Systems

Lecture1: Introduction of DES Basics

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What is Discrete Event System

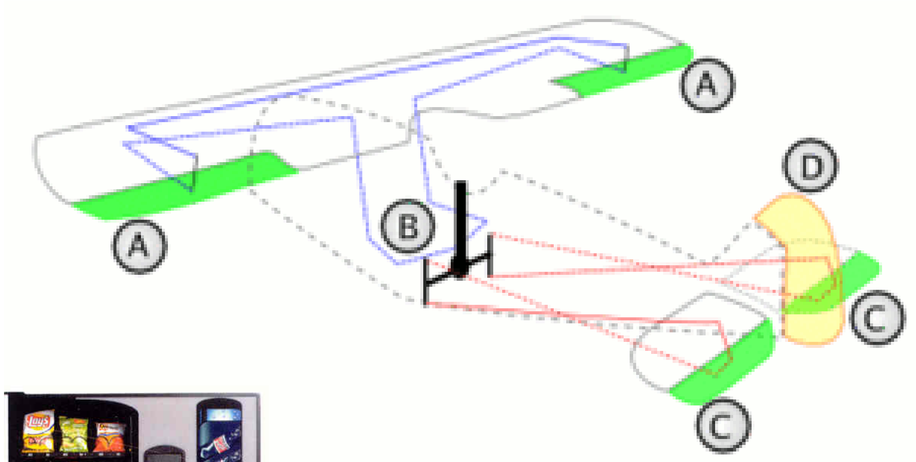
What is a system?

- A regularly interacting or interdependent group of items forming a unified whole (Webster dictionary).
- A combination of components that act together to perform a function not possible with any of individual parts. (IEEE Standard Dictionary of Electrical and Electronic Terms)
- A group of objects separated from the universe and having mutual relations. (textbook of Hruz and Zhou)

Examples of Systems

- **Flight Control System:**

  http://en.wikipedia.org/wiki/Aircraft_flight_control_system



- **Vending Machine:**

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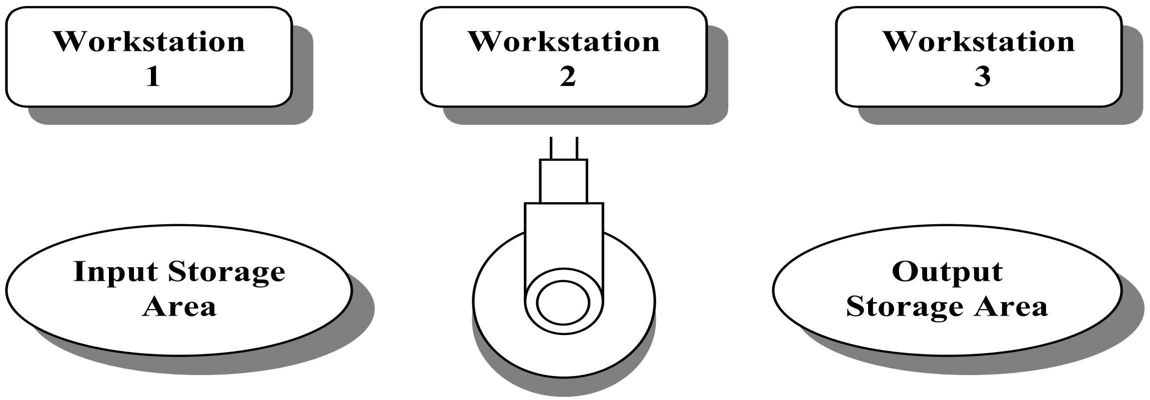
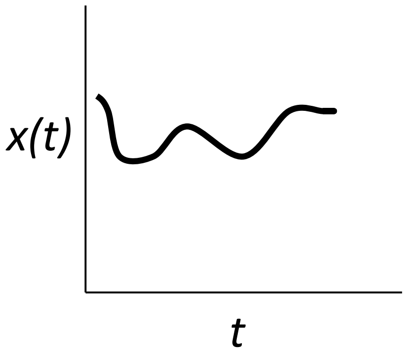
Examples of Systems

- **A manufacturing system with three workstations and one robot**

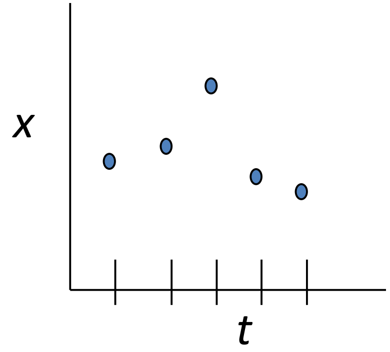


Continuous vs Discrete System

- A continuous system is specified by a set of continuous variables, functions and its derivatives.

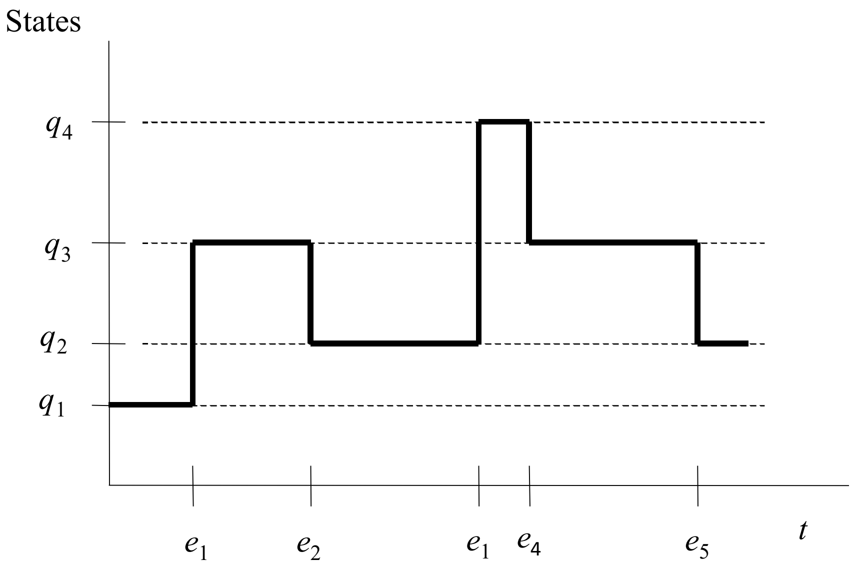


- A discrete system is specified by a set of discrete variables and relations defined on them.



Time vs Event Driven

- Discrete systems can be time-driven or event-driven.
- The discrete variable describes the state of the system.
- The following figure shows various states *q*1, *q*2, *q*3, … & different corresponding events *e*1, *e*2, and *e*3 … etc.
- This figure shows that the state change is event-driven.
- The events occur in discrete time points and the state changes depend only on the events. Such systems are called discrete event systems or DES for short



What are Discrete Event Systems?

- The systems are event-driven rather than clock driven, with state spaces that are discrete rather than continuous.

Examples of DES

- Light Switch

- Vending Machine

- More examples on next page ..

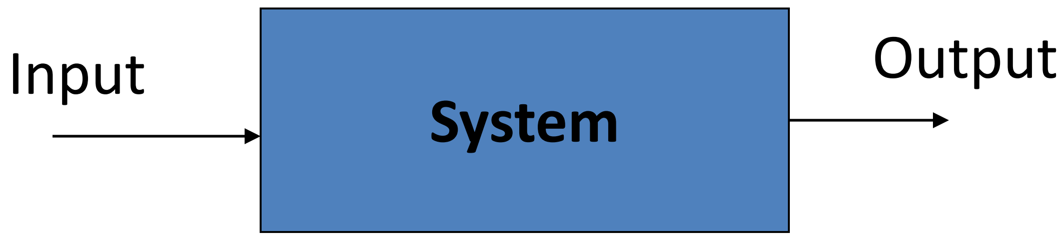
More examples of DES

- Queue systems: arrival, queue, server, departure
- Communication systems: idle, transmitting, collision/waiting
- Manufacturing processes: idle, loaded, processing, finished
- Traffic systems: intersections, traffic lights
- Computer: key pressed, mouse button clicked
- Database systems: read and write
- Lots of man-made systems: Microwave oven/Washing machine/Elevator …

Modeling DES

- We need to develop some mathematical means for describing the behavior of a system.
- Without such a model, it would be impossible to analyze and control DES just as it is true in classic control theory.
- A primary task of the DES theory is creating a DES model.
- We are interested in models based on which we can <u>design</u>, <u>analyze</u>, and <u>control</u> DES.

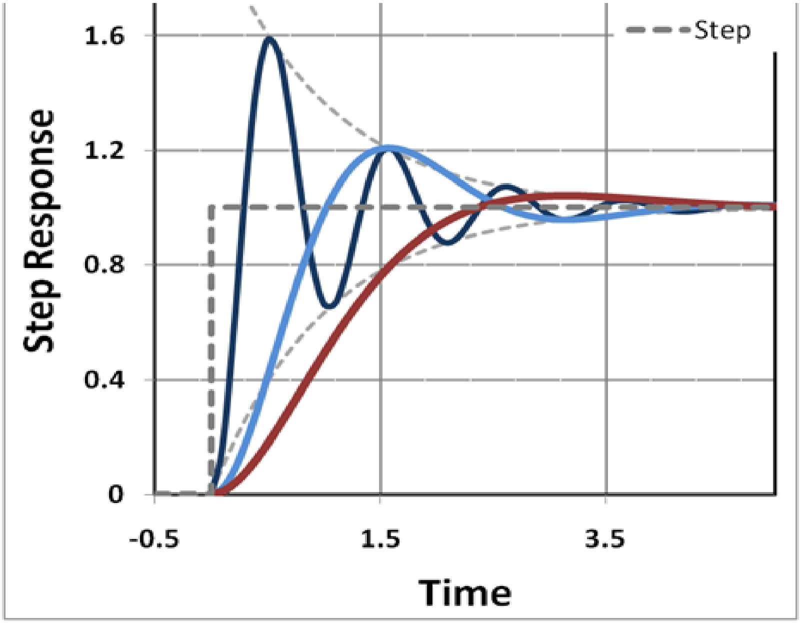
Modeling a System



e.g. A continuous system:

$$x'(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

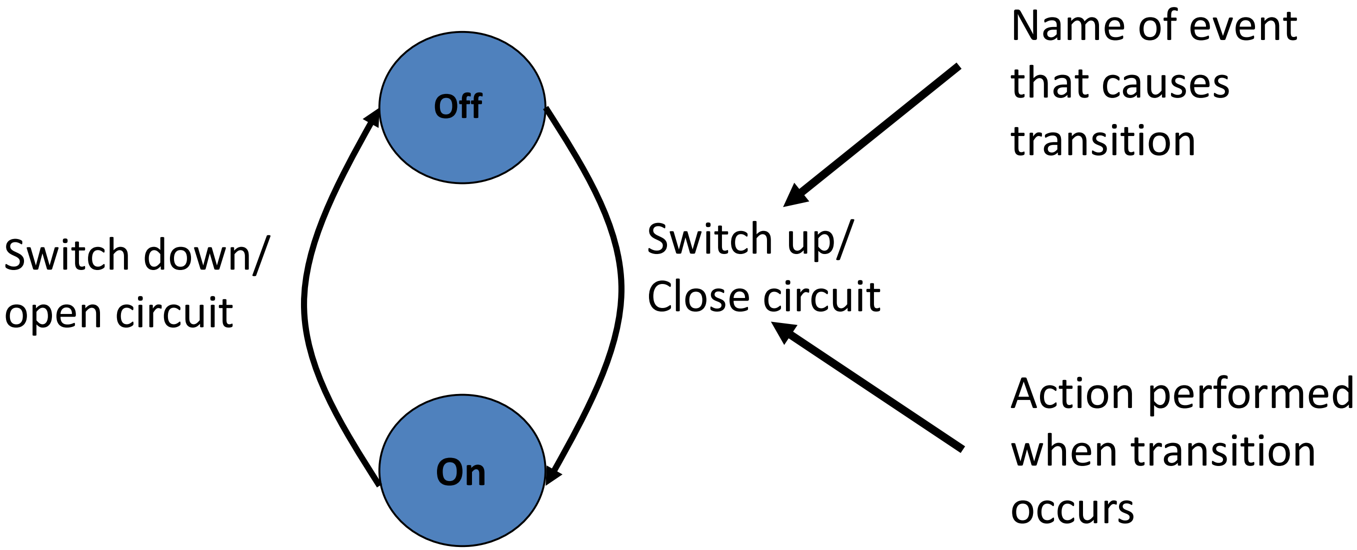
Three criteria particularly concerned in continuous system



- Stability
- Accuracy
- Rapidity

Examples of DES Modeling

- **Light Switch**

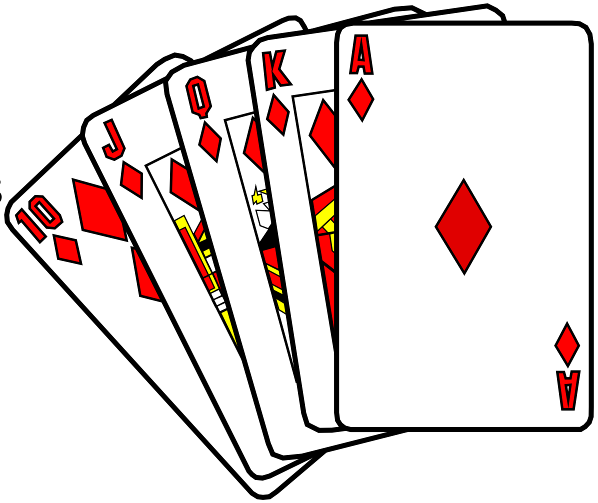


DES Modeling Basics

- **Set Theory -- *Definition***
  - A set is a collection of elements
  - An element is an object contained in a set
  - If every element of Set *A* is also contained in Set *B*, then Set *A* is a subset of Set *B*
  - The universal set contains all of the elements relevant to a given discussion
  - Examples:
    - The set is a deck of ordinary playing cards
    - Each card is an element in the set
    - Some subsets are:

      --- numbered cards

      --- suits



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DES Modeling Basics

- **Set Theory (cont.) -- *Notation***
  - Sets have been defined using the curly brace notation "{ … }" or descriptively "the set of all integers".
    e.g., { x | x is an even integer } ⟺ { $2x$ | x is an integer }
  - Notation

Symbol	*Meaning*
Upper case	designates set name
Lower case	designates set elements
{ }	enclose elements in set
$\in$ or $\notin$	is (or is not) an element of
$\subseteq$	is a subset of (includes equal sets)
$\subset$	is a proper subset of
$\not\subset$	is not a subset of
$\supset$	is a superset of
| or :	such that (if a condition is true)
| |	the cardinality of a set
$\forall$	Universal qualification – for all
$\exists$	Existential qualification – there exist

DES Modeling Basics

- **Set Theory (cont.) – *Set Equality***
  - Two sets are *equal* if and only if they contain precisely the same elements.
  - The order in which the elements are listed is unimportant.
  - Elements may be repeated in set definitions without increasing the size of the sets.
  - *Examples:*

$A = \{1, 2, 3, 4\} \quad B = \{1, 4, 2, 3\}$

$A \subset B$ and $B \subset A$; therefore, $A = B$ and $B = A$

$A = \{1, 2, 2, 3, 4, 1, 2\} \quad B = \{1, 2, 3, 4\}$

$A \subset B$ and $B \subset A$; therefore, $A = B$ and $B = A$

DES Modeling Basics

- **Set Theory (cont.) – *Cardinality***
  - Cardinality refers to the number of elements in a set
  - A *finite* set has a countable number of elements
  - An *infinite* set has at least as many elements as the set of *natural numbers*
  - notation: |A| *represents the cardinality of Set A*

Set Definition	*Cardinality*
A = {x | x is a lower case letter}	|A|=?
B = {2, 3, 4, 5, 6, 7}	|B|=?
C = {x | x is an even number $<$ 10 & >0}	|C|=?
D = {x | x is an even number $\leq$ 10 & $\geq$0}	|D|=?

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DES Modeling Basics

- **Set Theory (cont.) – *Power Set (𝒫)***
  - The power set is the set of all subsets that can be created from a given set
  - The *cardinality* of the power set is 2 to the power of the given set's cardinality. *If* $|A| = n$, then $|\mathcal{P}(A)| = 2^n$
  - notation: $\mathcal{P}$ (set name)
  - *Example:*

    $A = \{a, b, c\}$ where $|A| = 3$

    $\mathcal{P}(A) = \{\{a, b\}, \{a, c\}, \{b, c\}, \{a\}, \{b\}, \{c\}, A, \phi\}$

    $|\mathcal{P}(A)| = 8$

DES Modeling Basics

- **Set Theory (cont.) – *Set Operations***
  - Set theoretic operations allow us to build new sets.
    For example, given sets *A* and *B*, the set theoretic operators
    - Union ($\cup$)
    - Intersection ($\cap$)
    - Difference ($-$)
    - Complement ("—")
    - Symmetric Difference ($\oplus$)

    give us new sets $A \cup B$, $A \cap B$, $A - B$, $A \oplus B$, and $\bar{A}$.

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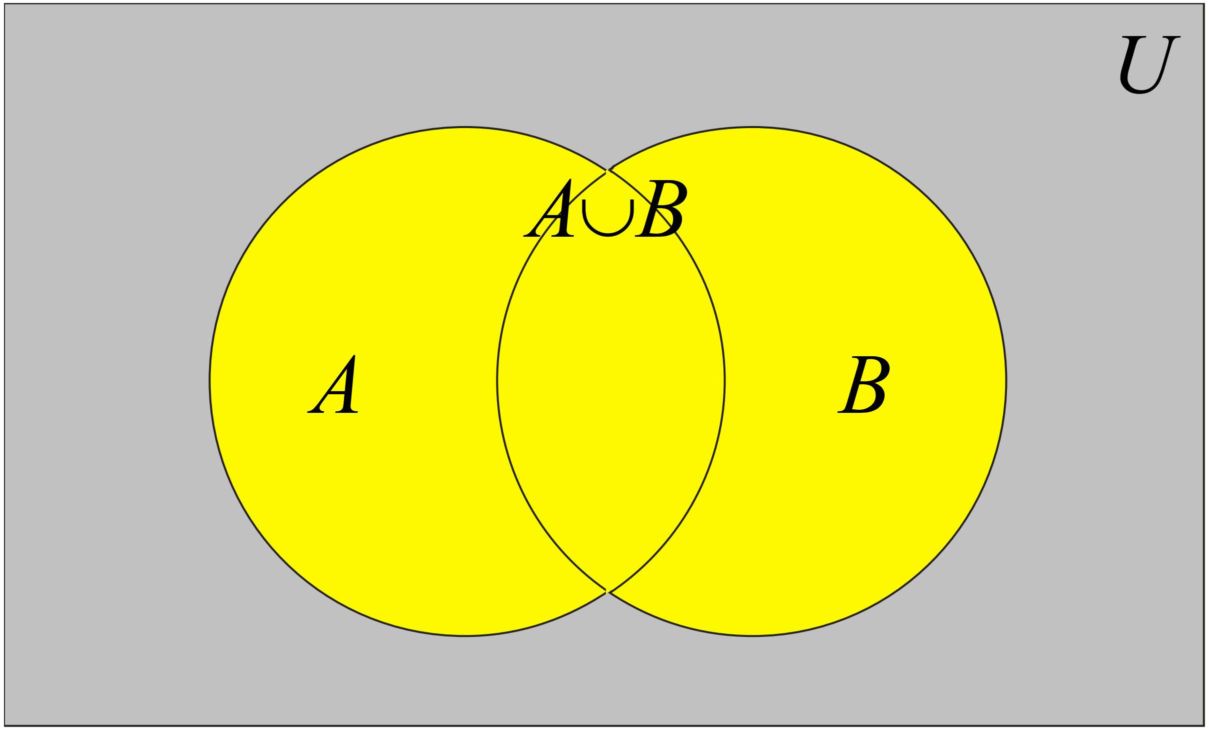
- **Set Theory (cont.) – *Set Operations -- Union***

Elements in at least one of the two sets:

$$A \cup B = \{ x \mid x \in A \vee x \in B \}$$



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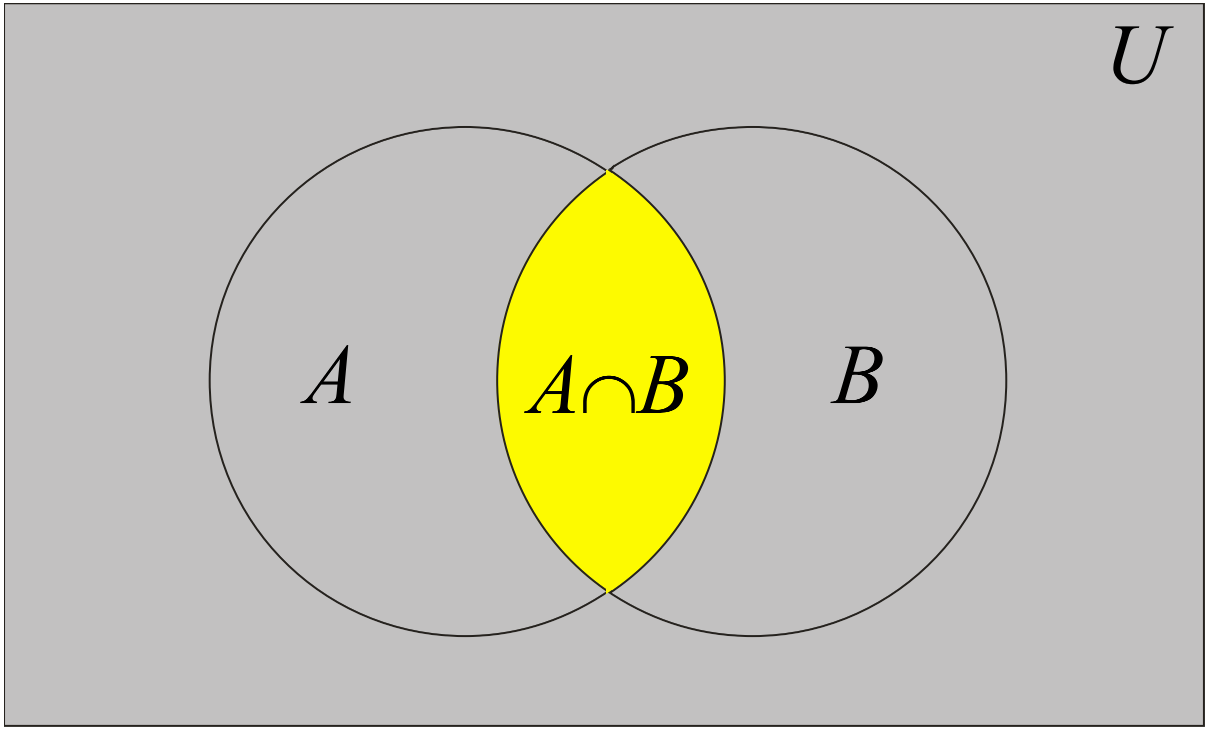
- **Set Theory (cont.) – *Set Operations -- Intersection***

Elements in exactly one of the two sets:

$$A \cap B = \{ x \mid x \in A \wedge x \in B \}$$



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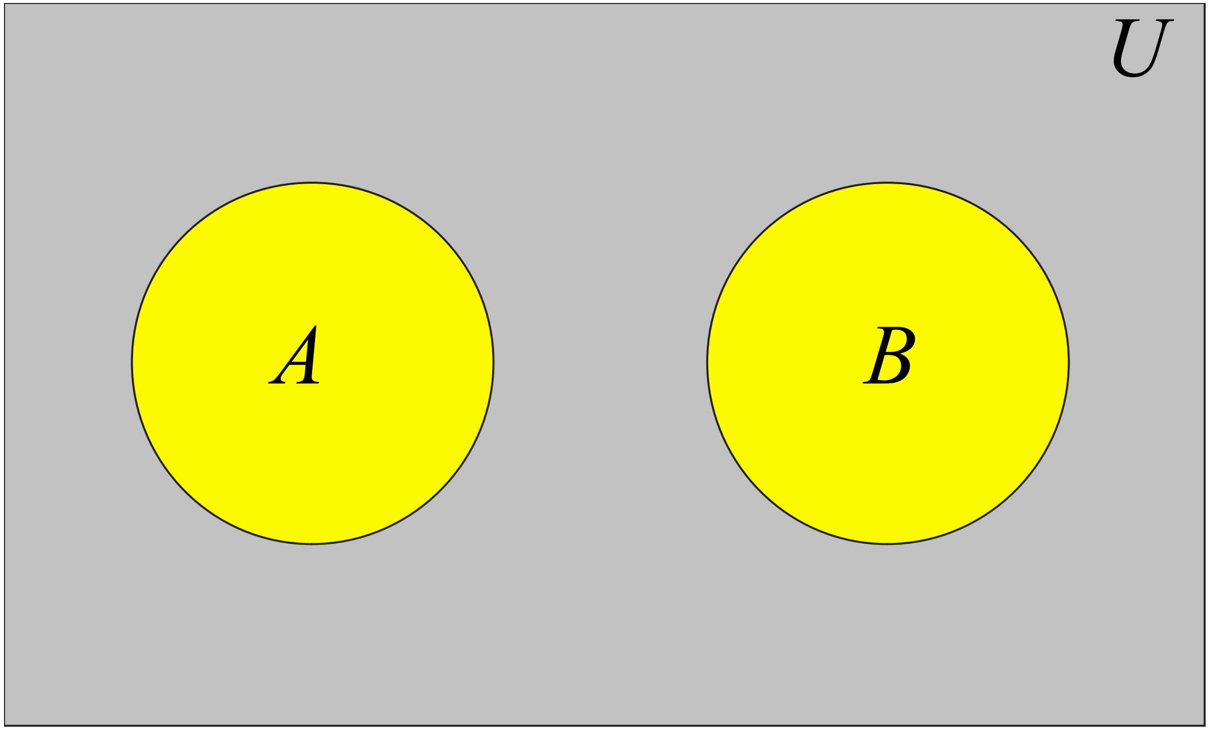
- **Set Theory (cont.) – *Set Operations -- Disjoint***

DEF: If *A* and *B* have no common elements, they are said to be ***disjoint***, i.e. $A \cap B = \varnothing$.



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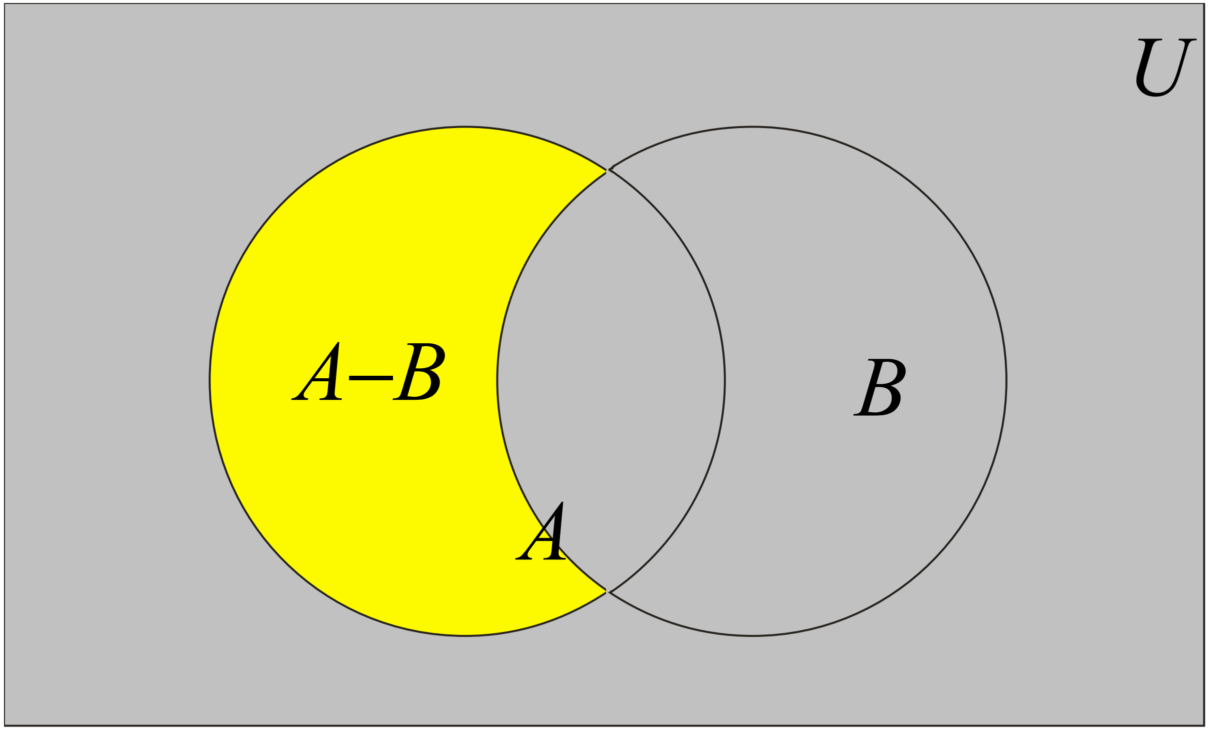
- **Set Theory (cont.) – *Set Operations – Set Difference***

Elements in first set but not second:

$$A - B = \{ x \mid x \in A \wedge x \notin B \}$$



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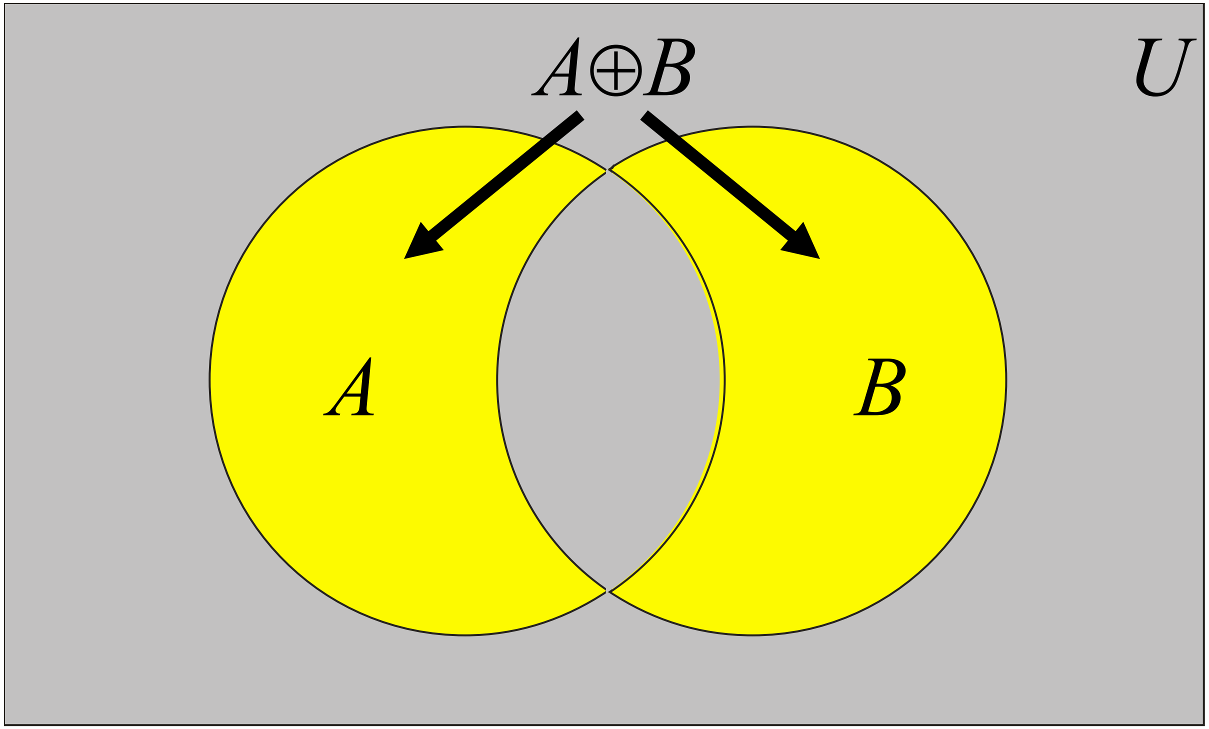
- **Set Theory (cont.) – *Set Operations – Symmetric Difference***

Elements in exactly one of the two sets:

$$A \oplus B = \{\, x \mid x \in A \;\oplus\; x \in B \,\}$$



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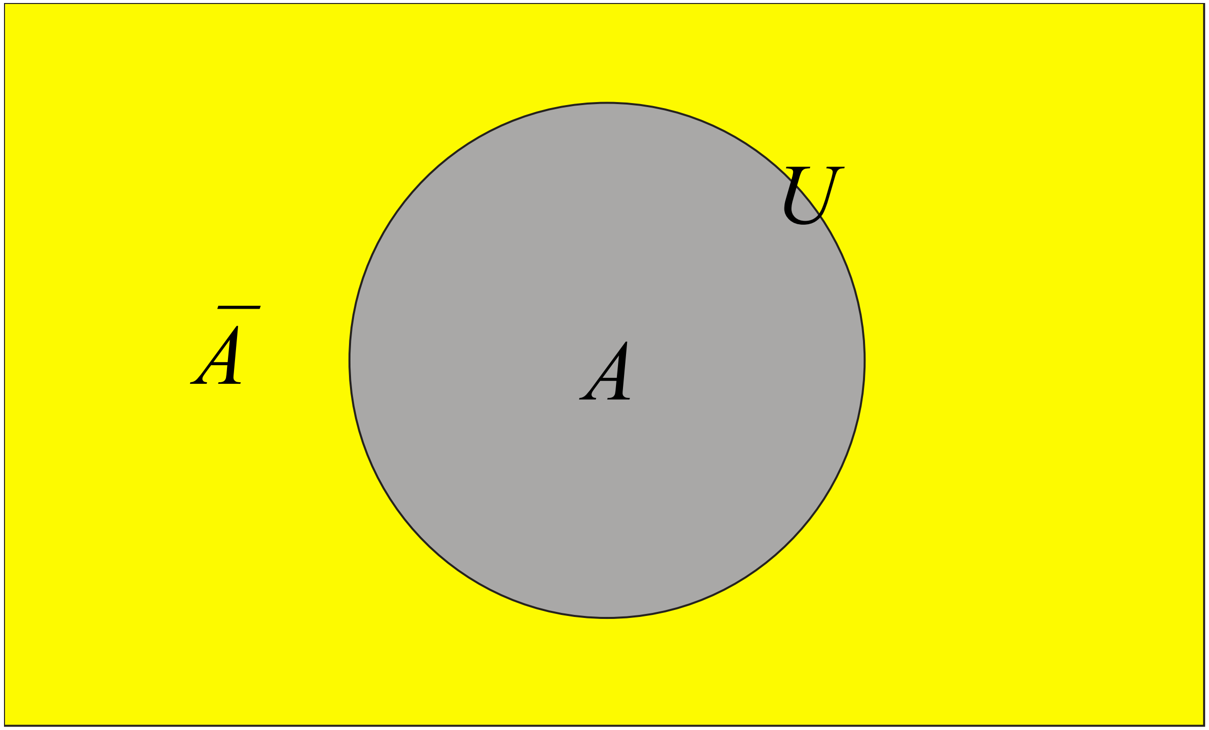
- **Set Theory (cont.) – *Set Operations – Complement***

Elements not in the set (unary operator):

$$\bar{A} = \{ x \mid x \notin A \}$$



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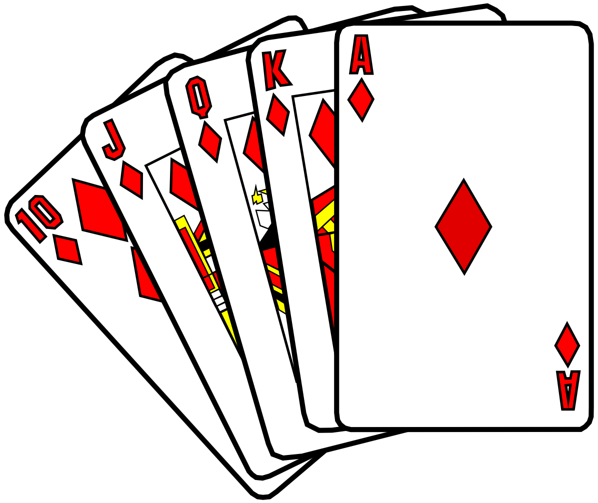
- **Set Theory (cont.) – *Binary Relation***

  - A binary relation R from D into E is defined by:

$$R \subseteq D \times E$$

    where symbol × denotes the Cartesian product

  - Examples:
    - $Ranks \times Suits$
    - $\text{Suits} \times Ranks$



  - R is a binary relation on a set *A* when D=E=A

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DES Modeling Basics

- **Discrete Process Representation**
  - ❑ Event set is defined as: □ $= \{e_1, e_2, e_3, \dots, e_n\}$
  - ❑ Assuming an event e_{i_k}□ occurs at the time point τ_{i_k}, a sequence of events, called a discrete process, can then be given as:

$$\tilde{\sigma} = e_{i_1}, e_{i_2}, \dots, e_{i_k}, \dots, e_{i_N}$$

where e_{i_1}□ occurs in the discrete time point τ_{i_1},
e_{i_2}□ occurs in the discrete time point τ_{i_2}, etc.,
e_{i_k}□ occurs in the discrete time point τ_{i_k}, etc., and
e_{i_N}□ occurs in the discrete time point τ_{i_N},
$\tau_{i_1} \langle \tau_{i_2} \langle \dots\dots \langle \tau_{i_k} \langle \dots\dots \langle\ \tau_{i_N}$.

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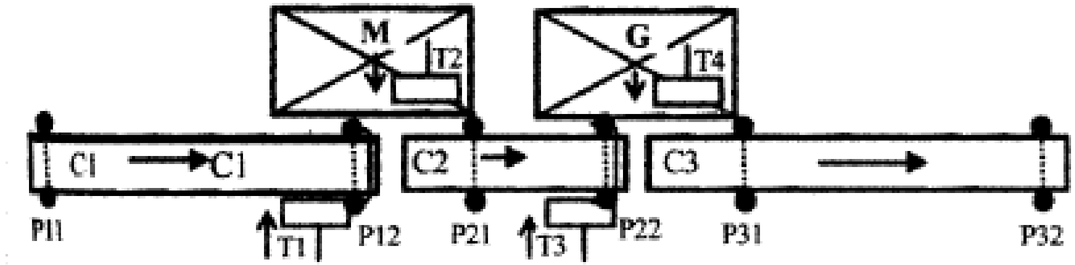
DES Modeling Basics

- **Discrete Process Examples**



➢ $\tilde{\sigma} = s_{c_1} e_{c_1} m_{c_1} s_M s_{c1} e_M m_{c_2} s_{c_2} e_{c_1} m_{c_1} s_M e_{c_2} g_{c_2} s_G$

➢ $\tilde{\sigma} = s_{c_1} e_{c_1} e_{c_2}$

➢ $\tilde{\sigma} = s_{c_1} e_{c_1} m_{c_1} s_M s_{c1} e_{c1} m_{c_1}$

Are those event sequences feasible? Admissible?

DES Modeling Basics

- **Basic transition system:**

$$SYST = (\Pi, \Sigma, Q, \Theta)$$

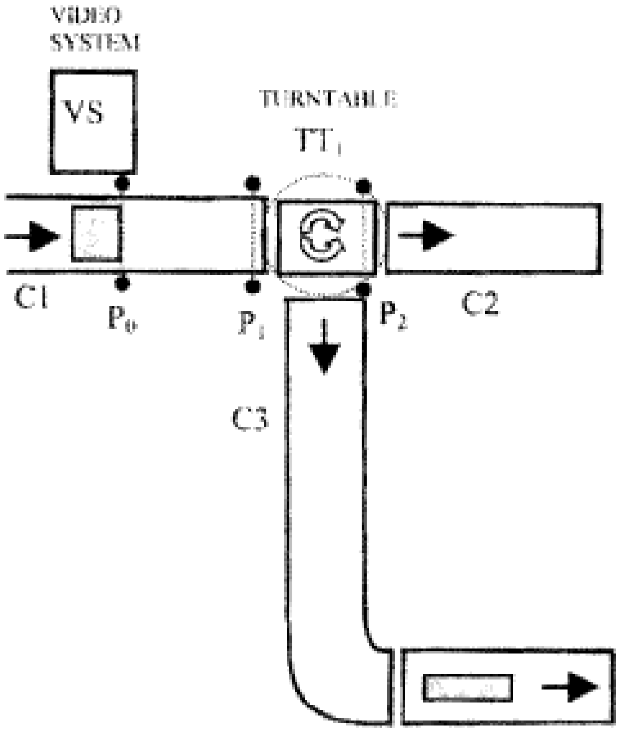
  - ❑ $\Pi = \{u_1, u_2, \ldots, u_n\}$ is the finite set of state variables
  - ❑ Q is the set of states where each state is given by the particular values of the variables from set
  - ❑ Σ is the set of transition whereby a transition $e \in \Sigma$ is a partial function $e : Q \xrightarrow{C_e} 2^Q$. Note that 2^Q is a set of all subsets of Q, and C_e is a condition imposed on a transition *e*
  - ❑ Θ is the set of initial conditions of the system. It includes states in which the execution of the potential events can start

DES Modeling Basics

- **Example:**

$$SYST = (\Pi, Q, \Sigma, \Theta)$$

$$\Pi = \left\{ \begin{matrix} P_0, P_1, P_2; \gamma_{01}, \gamma_{02}; T1; ETT1H, ETT1V; \\ TT1H, TT1V \end{matrix} \right\}$$



$$
\begin{aligned}
Q = \{ q_0 &= (0,0,0;\ 0,0;\ 0;\ 1,0;\ 1,0), & q_{12} &= (0,0,1;\ 0,1;\ 1;\ 0,1;\ 0,1), \\
q_1 &= (1,0,0;\ 0,0;\ 0;\ 1,0;\ 1,0), & q_{13} &= (0,0,0;\ 0,0;\ 0;\ 0,1;\ 0,1), \\
q_2 &= (1,0,0;\ 1,0;\ 0;\ 1,0;\ 1,0), & q_{14} &= (1,0,0;\ 0,0;\ 0;\ 0,1;\ 0,1), \\
q_3 &= (0,0,0;\ 1,0;\ 0;\ 1,0;\ 1,0), & q_{15} &= (1,0,0;\ 0,1;\ 0;\ 0,1;\ 0,1), \\
q_4 &= (0,1,0;\ 1,0;\ 0;\ 1,0;\ 1,0), & q_{16} &= (0,0,0;\ 0,1;\ 0;\ 0,1;\ 0,1), \\
q_5 &= (0,0,0;\ 1,0;\ 1;\ 1,0;\ 1,0), & q_{17} &= (0,1,0;\ 0,1;\ 0;\ 0,1;\ 0,1), \\
q_6 &= (0,0,1;\ 1,0;\ 1;\ 1,0;\ 1,0), & q_{18} &= (0,0,1;\ 0,1;\ 1;\ 0,1;\ 0,1), \\
q_7 &= (1,0,0;\ 0,1;\ 0;\ 1,0;\ 1,0), & q_{19} &= (1,0,0;\ 1,0;\ 0;\ 0,1;\ 0,1), \\
q_8 &= (0,0,0;\ 0,1;\ 0;\ 1,0;\ 1,0), & q_{20} &= (0,0,0;\ 1,0;\ 0;\ 0,1;\ 0,1), \\
q_9 &= (0,1,0;\ 0,1;\ 0;\ 1,0;\ 1,0), & q_{21} &= (0,1,0;\ 1,0;\ 0;\ 0,1;\ 0,1), \\
q_{10} &= (0,0,0;\ 0,1;\ 1;\ 1,0;\ 1,0), & q_{22} &= (0,0,0;\ 1,0;\ 1;\ 0,1;\ 0,1), \\
q_{11} &= (0,0,1;\ 0,1;\ 1;\ 1,0;\ 0,1), & q_{23} &= (0,0,1;\ 0,1;\ 1;\ 1,0;\ 1,0)\}
\end{aligned}
$$

DES Modeling Basics

- **Example (cont.):**
  - State transitions:

$$e_1 : e_1(q_0) = q_1,\ \ e_2 : e_2(q_1) = q_2,\ \ e_3 : e_3(q_2) = q_3,\ \ e_4 : e_4(q_3) = q_4$$

$$\begin{aligned}
\rho_4(\Pi,\Pi') = {} & (C1 \wedge C2 \wedge C3 \wedge INIT) \wedge \left(P_0' = lex_1 \text{ if and only if } lex_2 = 1\right) \wedge \\
& \left(P_1' = \overline{lex_1} \text{ if and only if } lex_2 = 1\right) \wedge \left(P_2' = \overline{lex_1} \text{ if and only if } lex_2 = 1\right) \wedge \\
& \left(\gamma_{01}' = \overline{lex_1} \text{ if and only if } lex_2 = 1\right) \wedge \left(\gamma_{02}' = \overline{lex_1} \text{ if and only if } lex_2 = 1\right) \wedge \\
& \left(T_1' = \overline{lex_1} \text{ if and only if } lex_2 = 1\right) \wedge \left(ETT1H' = lex_1 \text{ if and only if } lex_2 = 1\right) \wedge \\
& \left(ETTW' = \overline{lex_1} \text{ if and only if } lex_2 = 1\right) \wedge \left(TT1H' = lex_1 \text{ if and only if } lex_2 = 1\right) \\
& \wedge \left(TTW' = \overline{lex_1} \text{ if and only if } lex_2 = 1\right)
\end{aligned}$$

where $lex_1 = lex_2,$

$$lex_2 = \overline{P_0} \wedge \overline{P_1} \wedge \overline{P_2} \wedge \overline{\gamma_{01}} \wedge \overline{\gamma_{02}} \wedge \overline{T1} \wedge ETT1H \wedge \overline{ETTW} \wedge TT1H \wedge \overline{TTW}$$